

New Properties of Nash Correspondences: Externalities, Nonconvexities, and Fixed Points

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Abstract

We make two fundamental contributions to the theory of strategic form games with externalities. First, under mild conditions on the primitives, we show that for any collection of parameterized strategic form games, the equilibria-valued Nash correspondence defined on the parameter space (e.g., the space of externalities) is an upper semicontinuous mapping with nonempty, compact values (i.e., is an USCO). More importantly, we show that *all* minimal USCOs contained in the Nash correspondence are USCOs with connected values consisting of essential Nash equilibria in the sense of Fort (1950). Second, having established that all Nash correspondences contain minimal USCOs with essential connected values, we show that this implies that all Nash correspondences can be graphically approximated by continuous functions. As a consequence, we are able to show, for the case where the parameter space and the space of strategy profiles are equal, that all Nash correspondences - despite not having convex values or even connected values - have fixed points. Thus, we prove a new fixed point theorem for Nash correspondences, and using this theorem we establish the existence of *fulfilled expectations Nash equilibria* for a large class of strategic form games with externalities.

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