Echo Chambers:
Voter-to-Voter Communication and Political Competition[†]

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ABSTRACT

The rise of social media technologies has greatly increased the amount of voter-to-voter communication that is present in modern political systems. I investigate, in a model of informative campaign advertising, how the ability of voters to strategically communicate with each other shapes the advertising strategies of two competing parties. Two main results are put forward. First, information does not travel among voters biased toward different parties even if they are ideologically close – "echo chambers" arise endogenously. Second, whenever like-minded voters do not communicate with each other often enough (low homophily), parties tailor their advertising on their opponent's supporters rather than on swing or core states voters.

KEY WORDS: Cheap Talk, Political Advertising, Voter Communication

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1 Introduction

The recent emergence of social media technologies has greatly increased the communication flows that occur among voters in modern political systems. In turn, by mediating the exposure to information, these social media technologies have opened up an unexpected role for voters - they now not only cast their votes but also have a greatly enhanced role in spreading information that can subsequently shape general public opinion. Moreover, the combined use of social media and access to detailed personal information through big databases now gives political parties an unprecedented ability to individually target their political messages based on information about the known 'types' of voters.

Recent empirical studies have provided evidence on the decision-making processes that occur within voter communication networks (McClurg, 2003; Plutzer and Zipp, 1996; and Rainie and Smith, 2012) [1]. They find that interactions within particular types of networks, such as those based on intimate social relations, are very likely to foster a greater collective interest in politics. Simply put, social contact matters for the collection of political information among voters.

Another branch of research has then explored how the composition of the information that is gathered might matter. While individuals are arguably now overexposed to information and have access to virtually any information source, new technologies also allow them to restrict their exposure to like-minded sources, this way creating potential 'echo chambers' of political information. Earlier studies that focused on the consumption of 'top-down' news (eg: print newspapers, TV, online news websites) found muted evidence of higher information segregation (Gentzkow and Shapiro

[1] Approximately 80% of American adults use the internet and 66% of those online use social network sites (SNS) such as Facebook, LinkedIn, or Google+. Rainie and Smith (2012) find that about 75% of social network users in U.S. tend to share posts related to politics and political news with their contacts. And most importantly they find that the 25% of SNS users say they have become more active in politics as a result with 16% of SNS users saying they have changed their political views after discussing it or reading posts about it on the sites. McClurg (2003) shows that political interaction and social interaction are not independent, and interactions within intimate social network (between spouses, for example) increases the likelihood of political participation.

2011). However, recent studies focusing on social media point to stronger patterns of segregation. Bakshy, Messing, and Adamic (2015) investigate how U.S. Facebook users interact with socially shared news and examine the extent to which heterogeneous friends could expose individuals to cross-cutting content (that is, material that is not in line with their existing biases)[2]. They find that individuals' sharing choices played a stronger role than the Facebook content recommendation algorithm in limiting exposure to cross-cutting content. In this case, social contact matters, playing the role of an information filter for potential voters, but the potential theoretical basis of this phenomenon has not been investigated yet. This controversial paradox regarding the new information environment raises important questions about the potential effects of information exposure on political outcomes. Clearly, this information environment suggests that new approaches may be needed for the study of political competition amongst parties. Specifically, these approaches arguably need to combine the classic top-down model, in which parties use advertising to persuade voters, with a "bottom-up" model that takes into account the new role of voter-to-voter communication in shaping public opinion and influencing voting decisions.

In this paper I explore the mechanisms by which the communication flows that are facilitated by contemporary social media networks affect both voters' communication strategies and parties' advertising strategies.

I focus on a political game in which two policy-motivated parties compete in an election. Each party selects, given its candidate's type (extremist or moderate), the level of informative advertising it will use to persuade voters. I assume that parties' advertisements are truthfully informative but costly, and that each party seeks to maximise its' utility by winning the electoral competition. Voters have preferences over policies but they do not know ex-ante the ideological position of the parties' candidates. Voters can obtain information about parties' candidate type either by di-

[2]Facebook reflects the off-line social context of the users, that is, on-line links are mostly based on off-line relations rather than on common interests as in the case of blogs and other social media as Twitter. Thus the level of homophily within users' networks is reduced and 'real-world' personal relations are more likely among users connected to each other (Pew Research Center, Survey March 7 - April 4 2016, "Social Media Update").

rect exposure to the parties' advertisements or by communicating with other voters within their network. Voters update their beliefs based on the information gathered and cast their vote sincerely. The party that obtains the simple majority of votes wins the election and implements its policy.

A key feature of my model is that I relax the assumption of truthful communication between voters. This creates new strategic considerations for both voters and parties. Specifically, when voters can strategically communicate in this way they face the following trade off: on the one hand, if voters truthfully communicate their information to others they may increase the probability that a moderate candidate is elected. On the other hand, if voters lie about the good characteristics of the candidate they like less, they increase the probability that a moderate candidate of their favoured party wins the electoral competition.

As a practical example consider the case of left-right or Democrat-Republican competition with two candidates (Obama and Romney). There are two voters who are matched as sender and receiver, and the receiver's vote is pivotal. Assume both voters want to elect a moderate candidate, but they will always prefer to elect a moderate candidate who is affiliated to their favoured party.

Following this, suppose that the sender knows that Romney is a moderate but she has no information about the type of Obama. Then, a Democrat sender matched with Democrat receiver will always truthfully reveal her information about both candidates. In fact a Democrat receiver will vote for the moderate Romney if and only if she has no information at all about Obama's type, but will vote for Obama otherwise. However, if a Democrat sender is matched with a Republican receiver and she truthfully reveals her information, then the Republican receiver will always vote for Romney even if she knows that Obama is a moderate. By "lying" to the Republican receiver, and not revealing Romney's type, a Democrat sender may therefore increase the probability that the receiver will vote for a moderate Democratic candidate (Obama). Hence, Democrat-biased voters truthfully convey their information to other Democrat-biased voters, but will not convey any information otherwise. In short, Democrats only talk (truthfully) to Democrats and similarly Republicans only talk to other Republicans.

Thus when voters can strategically communicate with each other in this way echo chambers emerge endogenously: in equilibrium, voters only share valuable information with like minded peers that are biased toward their favoured party. In turn, this prevents the acquisition of valuable information among voters biased toward other party. The rise of the echo chambers, via the restricted diffusion of information, highlights how information flows can impinge on electoral competitions. In turn, this makes it crucial for parties to design their advertising strategies in a way that exploits these echo chambers so as to increase their probability of winning elections.

In line with this example, the first result of my model shows that voters value only information received from like-minded voters and disregard the information received from voters ideologically biased for the party they like less. This occurs regardless of the magnitude of the voter's biases in the matched pair. In other words, the ideological distance between voters during the communication stage does not play any role. The main determinant of the outcome of the communication stage is the direction of voters' biases and this leads to the emergence of echo chambers. Thus, when the probability that voters biased toward the same party interact is low (ie:low homophily) strategic information transmission entails a waste of valuable information. This result drives the second finding: if parties have access to targeting technologies, parties may prefer to target the voters that are ideologically biased for their opponent, rather than their own supporters, and exploit the echo chambers effect.

My model builds on the framework used by Galeotti and Mattozzi (2011), which investigates the effects of social learning on political outcomes in a model of truthful interpersonal communication and informative campaign advertising. They find that richer communication networks lead to political polarization, and they show that when parties can target their advertisements to groups with different ideological biases, parties always target the closer ideological group. My paper differs from Galeotti and Mattozzi's (2011) work in one important dimension: I analyse a political game of strategic information transmission among voters. While this setting increases the degree of misperception among voters, it also raises interesting questions regarding the targeting of advertisements and how the network's structure affects parties' incentives to disclose information.

The theoretical contributions advanced by the literature on communication flows and the effects of informative advertising on political competition can be summarized in terms of two main strands. The first type of arguments focuses on the effect of personal influence on electoral outcomes and voting decisions. When voters have similar preferences information transmission affects the outcome of the collective decision (Austen-Smith, 1990), but when voters have dissimilar or conflicting preferences information transmission is used as a double-check mechanism and players vote according to the information they hold only if the private signal is confirmed by the message received.

A second strand analyses the effects of informative advertising on political competition. There is not yet a consensus in this literature: while Cox and McCubbins (1986) find that parties tend to focus on "core supporters" when selecting targeted transfers strategies, Lindbeck and Weibull (1987) and Schulz (2003) predict that parties are more likely to spend resources for targeting "swing voters". Despite the relevant insights of these different approaches, the debate on the mechanisms of communication flows between parties and voters is still open. My contribution to this literature is in two fold: 1) I depart from the classical approach, which until now, considered voter's with similar preferences and I analyse how communication among voters, whose preferences are not necessarily similar affect the political outcome; 2) I stress the role played by interpersonal influences and social network's structure on parties' advertising strategies that has been little examined.

The remainder of the paper is organized as follows. Section 2 introduces and describes the model. Section 3 describes and characterizes the communication game. Section 4 provide the characterization of the political equilibrium. The last section concludes. All proofs are relegated to the appendices.

2 The Model

There are three types of players: two policy motivated parties that compete in elections, L party and R party, and a continuum of citizens of unit measure who have preferences over candidates (policy outcomes). The policy space is one-dimensional.

Nature independently and simultaneously assigns to each party a candidate type $t_J \in T \subseteq [0, 1]$, with $J \in \{L, R\}$, where a candidate type corresponds to a position on the policy space[3]. Let $\sigma : T \to [0,1]$ denote the probability distribution over candidate types, and let $\sigma(t_J)$ represent the probability that Nature assigns a candidate of type t_J to party $J \in \{L, R\}$. The probability distribution over candidate types is common knowledge. Following Galeotti and Mattozzi (2011), I restrict the candidates' type space for each party to $T = \{m, e\}$, where m and e denote respectively the positions of moderate and extremist types, where $e \equiv m/2$. A state of the world, then, is defined by the profile of types chosen by Nature, $\theta = (t_L, t_R) \in T \times T$.

Parties are asymmetrical informed about their candidate's type. Once both parties learn their candidates types, they simultaneously choose the level of informative campaign advertising, $x_J(t) \in [0,1]$. The advertising is truthful and fully informative[4], but costly. If a party chooses to advertise its candidate t_J with intensity $x_J(t_J)$, a random fraction $x_J(t_J)$ of voters will obtain complete information on the position of party J's candidate. Party L advertises a candidate of type t_L with intensity $x_L(t_L)$, where $x_L : T \to [0,1]$ to maximise its expected pay-off:

$$U_L(x \mid t_L) = \sum_{t_R \in \{e,m\}} \sigma_R(t_R)[\pi_L(x)((1-t_R)-t_L)-(1-t_R-e)] - cx_L(t_L) \tag{1}$$

where $\pi_L(x)$ denotes the expected probability that party L wins the election, $x = [x_L(t_L), x_R(t_R)]$ denotes the advertising levels chosen by both parties given their candidates' types, and $c \geq 0$ is the advertising unit cost.

Voters' payoffs are given by

$$u_i(t) = \begin{cases} t_L - i & \text{if } L \text{ wins the elections} \\ i - (1 - t_R) & \text{if } R \text{ wins the elections} \end{cases} \tag{2}$$

where $i \in (0,1)$ denotes voter's ideological bliss point, t_L denotes the ideological bliss point of the left party's candidate, and $(1 - t_R)$ the ideological bliss point of the right

[3]The candidate position on the policy space can also be interpreted as a measure of valence of the candidates, ex good Vs bad.

[4]The advertisements have two characteristics: 1) parties can only advertise their own candidate, i.e. I do not allow for negative advertising; 2) parties cannot lie about the ideological position of their candidates, by advertising their candidate they commit to implement the advertised policy if they win the election.

party's candidate, where $t_J = \{e, m\}$.

Voters can be divided in partisans and independent voters. Partisan voters, regardless of the information they hold, always vote for the party they are ex-ante biased toward to, while independent voters always prefers to vote for a moderate candidate rather than for an extremist one. In other words, if the state of the world is $\theta = (e, m)$, a perfectly informed independent voter will cast her vote for the right party regardless of her ex-ante ideological bias, while a perfectly informed right partisan will still vote for the the right party. Thus, the partisans' group ideology is distributed on the interval $(0, 1/2 - m/4)$ for the left partisans, and on $(1/2 + m/4, 1)$ for right partisans, and I assume the two groups have the same size [5]. The independent voters' ideological bliss points are uniformly distributed in the interval $[\mu + \tau, \mu - \tau]$, with $\tau > 0$, where μ is drawn from a uniform distribution with support $[1/2 - m/4, 1/2 + m/4]$. After the advertising has taken place, voters are randomly matched with a finite number, $k \geq 0$, of other voters and each sampled voter sends private messages, not necessarily truthful, to report the information obtained from parties' advertising. Within the sampled network of each voter there is no preference uncertainty: each voter knows the bliss point of all voters (both senders and receivers) with whom she is matched within her own network, but voters cannot observe the voting decisions of their peers. After voters communicate, the information of an independent voter with bliss point i becomes $I^i = (t_L, M_{k,L}(t_L), t_R, M_{k,R}(t_R))$, where t_J denotes the information about the candidate type of party J that the voter has acquired through direct exposure to the parties' advertisement, and $M_{k,J}(t_J)$ denotes the vector of messages received from her k partners about the candidate type of party J.

Denote by $\rho(t_L, t_R | I^i, x, k)$ the belief of an independent voter with bliss point i that candidates of the two party are respectively of type t_L and t_R, when she holds an information I^i, and the network richness is described by the parameter k. Whenever possible $\rho(t_L, t_R \mid I^i, x, k)$ is derived using Bayes' rule:

$$\rho(t_L, t_R \mid I^i, x, k) \quad = \quad \frac{\Pr(t_L, t_R) \Pr(I^i \mid t_L, t_R, k)}{\sum_{t'_L, t'_R \in T} \Pr(t'_L, t'_R) \Pr(I^i \mid t'_L, t'_R, k)}$$

Voters update their beliefs about the candidate's type of the two parties and sincerely

[5]This assumption is made only to simplify the mathematical model and avoid trivial results.

cast their vote. A voter with ideology i and information I^i votes for party L if and only if $i < i^*(I^i_{k+1})$, where $i^*(I^i_{k+1})$ is the bliss point of the indifferent independent voter with information I^i_{k+1}. Thus the identity of the indifferent voter for each information set I^i_{k+1}, is given by:

$$i^*(I^i_{k,L}, I^i_{k,R}) = \frac{1}{2} + \frac{\sum\limits_{t_L, t_R \in T} (\rho(t_L, t_R \mid I^i, x, k)t_L - \rho(t_L, t_R \mid I^i, x, k)t_R)}{2}$$

The left party gets at least half of the votes if and only if $\mu < \mu^*(x \mid \theta)$, where

$$\mu^*(x \mid \theta) = \sum_{I_{k+1}} i^*(I^i_{k+1}) \Pr(I^i_{k+1} \mid \theta, x, k)$$

Because the ideologies of independents is uniformly distributed in the interval $[1/2 - m/4, 1/2 + m/4]$, party L's candidate wins the election with probability

$$\pi_L(x_L \mid \theta, k) = \begin{cases} 0 & \text{if} \quad \mu^*(x \mid \theta) < \frac{1}{2} - \frac{m}{4} \\ \dfrac{\mu^*(x \mid \theta) + m - \frac{1}{2}}{2m} & \text{if} \quad \mu^*(x) \in \left(\frac{1}{2} - \frac{m}{4}, \frac{1}{2} + \frac{m}{4}\right) \\ 1 & \text{if} \quad \mu^*(x \mid \theta) > \frac{1}{2} + \frac{m}{4}. \end{cases}$$

The equilibrium concept is the *Perfect Bayesian Equilibrium*.

Definition 1 *A **Perfect Bayesian Symmetric Equilibrium** consists of (i) symmetric parties' advertising strategies, $x^* = (x^*_L(t_L), x^*_R(t_R))$; (ii) a pair of message strategy for each sender in each matched pair, $M_J(t_J) : T \to M$, for $J \in \{L, R\}$; (iii) voters' belief functions $\rho(t_L, t_R | (t_L, M_{k,L}(t_L), t_R, M_{k,R}(t_R)), k)$, indifferent independent voters $i^*(\cdot)$ such that:*

1) *$(x^*_L(t_L), x^*_R(t_R))$ are mutual best responses given the subsequent communication equilibrium and voting behaviour;*

2) *$i^*(\cdot)$ are consistent with $\rho^*(\cdot)$ and x^**

3) *$\rho^*(\cdot)$ are consistent and sequentially rational with x^*, with the senders' message strategies, and the voting decision.*

3 Communication in the Social Network

The first result shows that truthful revelation is possible if and only if both the sender and the receiver belong to the same group-type of voters (partisans or independent) and they are both ex-ante biased toward the same party. In other words, in each matched pair of ex-post independent voters, if both the sender and the receiver are ex-ante biased toward the same party there is always fully revealing perfect Bayesian equilibrium in any state of the world and this holds regardless of the distance between the biases of the two voters. Conversely, if two voters in a matched pair do not belong to the same group type, i.e. if they are not either both independent or both partisans, or they are ex-ante biased toward different parties then there is never a fully revealing perfect Bayesian equilibrium, for any small distance between their biases.

The social network is randomly determined. Three driving elements of the communication game. First, given that the two groups of partisans have same size, the independent voters are decisive for the outcome of the election. Thus I assume, without loss of generality, that independent voters communicate only with other independents voters[6]. Second, I assume there is no preference uncertainty within the sampled network of each voter. Last, I assume the network is acyclic[7].
These characteristics of the communication network allow each voter to evaluate the messages she receives one by one, then aggregate the information and update her beliefs.

To isolate the informational incentives I assume that each sender considers each of her receivers as if they were pivotal. This assumption formalizes the idea that

[6]Including partisans in the communication game will not affect qualitatively the results for two reasons: First, partisans' voting decisions are not affected by the information they acquire. A partisan, by definition, always votes for her favoured party regardless of her information about the two candidates. Second, in the communication stage, partisans have no incentive to share good information about the candidate of their less favoured party because it would increase the probability that her less favoured party wins the election.

[7]Allowing the information to travel for more than one step complicates the analysis, but it would not affect the qualitative features of my results given that in the equilibrium of the communication game the only network's feature that matter is the bias direction of the voters in each matched pair.

when voters share information with each other they believe they are able to affect the probability that their favoured candidate or party will win the election. Thus, in each matched pair of voters (where, abusing of notation, I denote by s and r the bliss points of the sender and the receiver respectively) the sender's expected payoff from revealing her information to the receiver is

$$\begin{aligned} u_s((M_L(t_L), M_R(t_R)) \mid I^s, x, k, r) &= \Pr[v_r = 1 \mid I^r, x, k, r]\mathbb{E}[t_L - s \mid I^s, x, k, r] \\ &+ \Pr[v_r = 0 \mid I^r, x, k, r]\mathbb{E}[s - (1 - t_R) \mid I^s, x, k, r] \end{aligned}$$

where I^s is the information held by the sender after the advertising has taken place, and $\Pr[v_r = 1 \mid I^r, x, k, r]$ is the probability that the receiver with bliss point r votes for the left party given her ex-post information I^r, the advertising strategy of the two parties, and the network structure, k. Note that the equation above depends on the information of both players, it implies that the informational incentive of the sender depends not only on her own information but also on both receiver's actual information and receiver's ideological bias. In the last stage, given the information I^i, the equilibrium parties' strategy x^*, and the network structure k, the voting decision of an ex-ante left biased voter i is

$$v_i = \begin{cases} 1 \text{ if } \mathbb{E}\left[t_L - i \mid I^i, x, k\right] \geq \mathbb{E}\left[i - (1 - t_R) \mid I^i, x, k\right] \\ 0 \text{ otherwise.} \end{cases}$$

Where the beliefs of voter i, about the two candidates are respectively of type t_L and t_R, $\rho(t_L, t_R | I^i, x, k)$ is derived using Bayes rule whenever possible. Thus, the receiver's expected pay-off is

$$u_r(v_r \mid I^r, x, k) = v_r\mathbb{E}[t_L - r \mid I^r, x, k] + (1 - v_r)\mathbb{E}[r - (1 - t_R) \mid I^r, x, k].$$

Truthful revelation is incentive compatible for the sender if and only if

$$u_s((M_L(t_L), M_R(t_R)) \mid I^s, x, k, r) \geq u_s((M_L(\hat{t}_L), M_R(\hat{t}_R)) \mid I^s, x, k, r) \tag{3}$$

assuming that the indifferent sender sends a truthful message.

In order to see the intuition behind the first result consider a scenario in which an ex-post independent left-biased sender knows the right party's candidate is a moderate and she does not hold any information about her favoured candidate's party.

Consider a scenario in which an ex-post independent left-biased sender knows the right party's candidate is a moderate and she does not hold any information about her favoured candidate's party. First, suppose the sender is matched with a receiver who prefers the same party as she. If the sender truthfully reveals her information, the receiver will vote for the party that both like less if and only if she has no credible information about their favoured party's candidate type. Otherwise, the receiver will always prefer to vote for the party that both she and the sender like most.

Suppose the sender is matched with a receiver who is biased in opposite direction, i.e. the two voters ex-ante prefer different parties. If the sender truthfully reveals her information, the receiver will always cast her vote for her favoured party disregarding any valuable information about the party's candidate she likes less. On the contrary, if the sender does not reveal her information about the candidate she likes less the receiver may use her own information, if she has any, about the left party's candidate and cast her vote for it. Given that there is no preference uncertainty, in each matched pair of voters the receiver can disregard all messages received from senders who are ex-ante biased for the party she likes less.

It follows that, if the voter receives at least one credible and informative message about the candidate's type of party J, say $M_J(m)$, then she knows that the J's candidate is a moderate with probability one, and regardless of the other $k-1$ messages she receives. Conversely, if the receiver gets k credible but non-informative messages about the candidate's type of party J, i.e. $M_J(\emptyset)$, then she knows that the candidate of party J is a moderate with probability $\sigma(m)(1-x_J(m))^k < 1$, and he is an extremist with probability $1-\sigma(m) > 0$.

Before stating the first theorem it is convenient to define some notation. In what follows I denote by $\beta \in (0,1)$ the probability that in each of the k matched pairs, both voters are ex-ante biased toward the same party, and by $x_J^*(t)$ the equilibrium level of party J advertising given its candidate type t. Thus the symmetric equilibrium is characterized by two cutoffs,

$$q_l = \frac{1}{2} - \frac{m}{4}\frac{1-\sigma_L}{1-\sigma_L+\sigma_L(1-x_L^*(m))^{\beta k+1}},$$

and

$$q_r = \frac{1}{2} + \frac{m}{4} \frac{1-\sigma_R}{1-\sigma_R+\sigma_R(1-x_R^*(m))^{\beta k+1}},$$

that depend on the network's richness k and the degree of homophily β. These cutoffs define the three types of voters, based on their voting behaviour, after the communication stage. Types with bliss point $i < q_l$ ($i > q_r$) always vote for the left (right) party regardless the information they hold, types with $q_l \leq i \leq q_r$ will cast their vote according to the information they hold.

Theorem 1 *In any pairwise k-players cheap-talk game there are two ideological cutoffs,* $0 < q_l < q_r < 1$*, such that truthful revelation is possible if and only if the bliss points of both voters in the matched pair belong to the same interval defined by those cutoffs and voters are ex-ante biased in the same direction. There does not exist any other configuration of the biases of voters such that truthful information transmission is possible.*

Theorem 1 says that two matched voters truthfully convey their information in the communication stage if and only if they belong to the same voter's type (partisan or independent) and they are ex-ante biased toward the same party, regardless of the distance between the two voters' biases. The two voters send a babbling message otherwise. The parameter β is a measure of the homophily of the network (for example, when $\beta = 0$ the sender and the receiver in each matched pair are biased toward different parties, while for $\beta = 1$ only voters that are ex-ante biased toward the same party communicate with each other).

4 Advertising Strategies and Political Equilibrium

In this section I analyse how the network's structure affects the advertising strategies of the two parties. In particular, I compare the maximum unit cost a party is willing to pay to advertise its candidate when the targeting technology is not available (i.e. a party can only randomly advertise their candidates), with the case in which a group-targeting technology is available. Further, I show how those choices are affected by both the homophily, β, and the richness of the network's structure, k.

Theorem 1 shows that if independent voters are allowed to strategically transmit their information, the information never travels between groups ex-ante biased toward different parties. To see how the network's structure affects the political outcome, it is helpful to look at the diffusion of the information in the population when parties randomly advertise their candidate. Given the advertising strategy of the party L for the candidate type t_L, $x_L(t_L)$, the fraction of voters that believe that the candidate of the left party is of type t_L, once the communication game takes place, is

$$z_L(x_L, k, \beta) = 1 - (1 - x_L(t_L))^{\beta k + 1},$$

i.e. the fraction of voters who are informed and trust their information about the candidate's type of party L depends on both the richness of the network k, and on the degree of homophily in the network β. These two variables play an important rule by affecting the information diffusion and, in this way, shaping the advertising and targeting strategies of the parties. The second result shows that, if a targeting technology is available, a party that observes a moderate candidate will never target its own supporters, but it may prefer to target the supporters of its opponent rather than randomly advertise its candidate depending on the network's structure.

Theorem 2 *In a symmetric pure strategy equilibrium parties never tailor their advertisements exclusively to their own supporters, and there exist critical levels of richness of the network, k, and degree of homophily, β, such that party L advertises a moderate candidate exclusively to the opponent's supporters if and only if the cost of advertisement is such that*

$$c(k, \beta) < \frac{(2 - 3m)(1 - \sigma_R)}{4},$$

it randomly advertises a moderate candidate if and only if

$$c(k, \beta) < \frac{(1 - \sigma_R)(\beta k + 1)\rho(m, e \mid I, \beta, k)(2 - 3m)}{16}$$

and never advertise a moderate candidate otherwise.

As Theorem 1 shows, information does not travel among groups of voters if they are allowed to strategically communicate with each other. Then it becomes crucially important for parties, when advertising, to consider how the network's structure determines their probability of winning the election by affecting the diffusion of good information. When the network shows an high degree of homophily (good information does not get disregarded in the communication stage) and/or the network is rich (in the communication stage voters are virtually able to access to many sources of information), parties prefer to randomly advertise their candidate. By randomly advertising their candidates parties make sure that independent voters of both ideological groups are exposed to good information with positive probability, and, by exploiting the echo chambers, they make good information reach an high number voters. In other words, through the communication stage the advertisement's persuasion power is amplified whenever degree of homophily and the richness of the network are high.

Formally, party L's expected utility from randomly advertising a moderate candidate is

$$\begin{aligned} U_L(x_L(m)|k,\beta) &= \sigma_R(m)\left[\frac{1}{2}(1-2m)-\frac{2-3m}{2}\right] \\ &+ (1-\sigma_R)\left[\frac{4+\rho(m,e\mid I,\beta,k)(1-(1-x_L(m))^{\beta k+1})}{8}\frac{2-3m}{2}\right. \\ &- \left.(1-m)\vphantom{\frac{1}{2}}\right]-cx_L(m) \end{aligned}$$

Its optimal level of advertisement $x_L^*(m)\in(0,1)$ solves

$$(1-\sigma_R(m))(\beta k+1)\rho(m,e\mid I,\beta,k)(1-x_L(m))^{\beta k}=\frac{16c}{2-3m} \tag{4}$$

Thus, a party randomly advertises a moderate candidate whenever the cost of advertisement is such that

$$c(k,\beta)\leq\frac{(1-\sigma_R(m))(\beta k+1)\rho(m,e\mid I,\beta,k)(2-3m)}{16}$$

and never it advertises a moderate candidate otherwise.

Conversely, a low degree of homophily and/or low richness of the network prevent good information to be spread among voters, given that good information is transmitted only among like-minded voters and uninformed independent voters always vote their favoured party. Thus, parties may prefer to tailor their advertisement to only one of the two groups of independent voters to exploit the echo chamber's effect if the voters' network shows low degree of homophily or richness of the network.

It is easy to show that parties never target the advertisement to their own supporters. In what follows, to simplify the argument, assume that if a party decide to target its advertisement to only one group of independent voters, then all voters that belong to that group are informed about the party candidate's type.

Suppose party L targets its advertisement on its own supporters. Because the information does not travel between the two groups of independent voters, good information does not reaches opponent's supporters and party L loses some of these votes. Since party R's supporters are uninformed about the candidate type of party L they never vote for party L, but they would have voted for it if informed. Moreover, party L, by targeting its own supporters also wastes the persuasive power of the advertisement: party L's supporters would have voted for it even if uninformed. Thus, by tailoring the advertisement on its own supporters a party wastes part of its persuasion power on those voters who would have voted for it even if they were not hit by the advertisement, instead of persuading opposite biased voters that would have voted for it if hit by the advertisement.

In other words, if party L targets its advertisement to its own supporters and both parties have a moderate candidate, then party L wins the election competition with probability $1/2$, i.e. the probability of winning the election is unaffected with respect to the case in which both parties randomly advertise their candidates. When the opponent's candidate is an extremist the probability of winning the electoral competition decreases from $\pi_L(m, e \mid x) = 1/2 + \rho(m, e \mid I, \beta, k)(1 - (1 - x_L(m))^{(\beta k+1)}$ (the probability of winning the election when the party advertises a moderate candidate randomly) to $\pi_L^l(m, e \mid y) = 1/2$ (the probability of winning the election when it advertises its candidate only to the its own supporter). It is crucial to notice that parties are ideologically motivated, this implies that parties would prefer to lose the electoral competition when the opponent has a moderate candidate (the less harmful case), and they prefer to win the election with probability one whenever the opponent has an extremist candidate (the most harmful state of the world). Then, parties always prefer to randomly advertise their candidate rather than target their own supporters to not waste the advertisement's persuasion power.

Parties can also target the advertisement on the opponent's supporters to increase the persuasion power among the voters who need more information to switch their vote, in this way party can also exploit the opponents' echo chambers to amplify the power of the information within that group of voters.

Suppose party L targets the advertisement on the opponent's supporters. On the one hand, if party L targets its advertisement on the supporters of party R then no left biased voter will have any information about the candidate of her favoured party. But a low degree of homophily implies that, if party R randomly advertises its candidate, party L's supporters are more likely to be uninformed about the candidate's type of party R and, then, vote for party L. On the other hand, by tailoring its advertisement on the opponent's supporters party L increases the probability that its own good information is spread among the R's supporters, i.e. among those voters that would vote for it only if sufficiently informed. This implies that, if the opponent's candidate is extremist, then the party wins the electoral competition with probability one. Thus, when a party has a moderate candidate, if the network's structure entails a big waste of information, i.e.

$$\frac{4}{\beta k+1} > \rho(m, e \mid I, k, \beta),$$

then the parties prefer to target their advertisement to the opponent's supporters whenever the cost of the advertisement is low enough, i.e. such that

$$c(k,\beta) < \frac{(2-3m)(1-\sigma_R)}{4}.$$

5 Discussion

Recent technological changes have lead to the rapid development of communication networks through social media, with political implications that are only beginning to be understood. In turn, this has created a major paradox for the standard model of political competition in environments where strategic information is important. Specifically, this paradox revolves around the fact that while new technologies have massively increased the availability of information to voters (via direct news, social media exchanges or targeted political advertising) they also seem to have facilitated the development of segregated information environments – so-called echo chambers.

These echo chambers are a potential problem because they prevent voters from acquiring as much information as they feasibly could and valuable information may be wasted or disregarded as a result.

To explore this paradox and to understand how social media are shaping both the communication strategies of voters and the advertising strategies of political parties I build a model based on a two level game. This overall model embeds (i) a model of campaign competition, where parties compete in campaign advertising (first level of the game), and (ii) a model of personal influence where voters, whose preferences are not necessarily similar, can strategically communicate with each other to affect the policy outcome (second level of the game). Importantly, I do not assume truthful communication of information between voters. In turn, this opens up new considerations for strategic information transmission between voters, leading to two new results.

The first result shows that, whenever voters can strategically communicate with each other, echo chambers rise endogenously and individuals value only the information received from like-minded voters, regardless of the bias gap. In other words, I show that the main determinant in the communication game is the direction of the biases and not the bias gap.

The second result suggests that both the richness of the network and the degree of homophily play an important role in the advertising strategies of the parties. Specifically, whenever either the richness of the network, or the degree of homophily within the network or both, are low, the parties are likely to tailor their advertising campaign to voters ideological biased toward their opponent - rather than targeting the closer ideological group of voters, as the literature with truthful communication suggests.

The crucial contribution of this paper is its ability to disentangle the mechanisms throughout communication flows within contemporary mass media networks shapes the electoral competition. The two results described above suggest that political interaction in the social network can not be ignored, neither can be ignored the role played by the network structure. For example, referring to two of the most famous social network sites, Facebook and Twitter, my results would suggest to adopt different advertising strategies. For Facebook, whose users are more likely to build networks based on their 'off–line' social interactions (that is users' network is less likely to show high homophily) parties should target their advertisement to opponent's supporters to increase their persuasion power and this way their probability of winning the electoral competition. Conversely, in the case of Twitter, whose users networks are build on common interests and/or specific content (i.e. users' network show high homophily) parties should randomly advertise their candidate to increase the persuasion power of the advertisement.

Understanding how social network shape voting decisions became crucially important also for the mobilization of the so-called independent voters, who are likely to base their voting decisions mostly on candidates' personal characteristics, that are decisive for election outcomes. So far, the role played by network structure has been comparatively little examined, but it is important to analyse how an endogenous mechanism of network formation and information acquisition may shape political mobilization and the electoral outcome.

A Appendix

Before proceeding with the proofs of the model's results notice that parties never advertise an extremist candidate. Consider the case in which there is only one independent voter whose vote is decisive for the election and there is no communication among voters. By way of contradiction, suppose the left party's candidate is an extremist and it is advertised with intensity $x_L(e) > 0$ while the right party's candidate is a moderate. Then if a voter with bliss point i is partially informed about the left candidate's type and parties play symmetric strategies, she will vote for the left party if and only if her bliss point is such that $i < \frac{1}{2} - \frac{m}{4}\rho(e, m \mid (e, \emptyset), x)$, with $\rho(e, m \mid (e, \emptyset), x) > 0$, that is only if she is an ex-post partisan. An uniformed voter, instead, will vote for the left party if and only if her bliss point is such that $i' < \frac{1}{2}$. This implies that if the left party does not advertise an extremist candidate, it increases her probability of winning the election. Therefore, parties will never advertise an extremist candidate. A party that advertises an extremist candidate bears two costs: a direct cost, given that the advertisement technology is costly, and an indirect cost due to disclosing harmful information that does not allow parties to exploit the erroneous beliefs of voters, a voter who is perfectly informed about J's candidate to be an extremist will more often vote for its opponent.

A.1 Communication Game

In what follows, I first study the senders' message strategies and the subsequent voting strategies of the receivers when the network richness is $k = 1$, that is when each voter is linked to only one sender, then I formally introduce a more complex communication network.

Two are the crucial characteristics of the communication network: there is no preference uncertainty within groups, that is each voter knows the bliss point of all the voters (both senders and receivers) she is matched with, and the communication among voters has the form of private messages. Then I can focus on pairwise communication strategies. Consider a matched pair of voters where, with some abuse of notation, I denote by s and by r the bliss points of the sender and the receiver, respectively. The sender's expected payoff from revealing her information to the receiver is

$$\begin{aligned} u_s((M_L(t_L), M_R(t_R)) \mid I^s, x, k, r) &= Pr[v_r = 1 \mid I^r, x, k, r]\mathbb{E}[(t_L - s) \mid I^s, x] \\ &+ Pr[v_r = 0 \mid I^r, x, k, r]\mathbb{E}[-((1 - t_R) - s) \mid I^s, x] \end{aligned}$$

where I^s and I^r denote the information set of the sender and the receiver after the communication game has taken place, and $Pr[v_r = 1 \mid I^r, x, k, r]$ is the probability that the receiver votes for the left party given her ex-post information set I^r, her bliss point r, and the network richness described by k. The informational incentive of the sender depends, not only on the information she holds, but also on both the receiver's actual information and receiver's ideological bias. The receiver's expected pay-off is

$$u_r(v_r \mid I^r, x, k) = v_r\mathbb{E}[t_L - r \mid I^r, x, k] + (1 - v_r)\mathbb{E}[-(1 - t_R - r) \mid I^r, x, k]$$

where $v_r = \{0, 1\}$ is the receiver voting decision. An ex-ante biased receiver will vote for party L if and only if

$$\mathbb{E}[(t_L - r) \mid I^r, x, k] \geq \mathbb{E}[-((1 - t_R) - r)) \mid I^r, x, k]. \tag{5}$$

Thus, truthful revelation is incentive compatible for the sender if and only if

$$u_s((M_L(t_L), M_R(t_R)) \mid I^s, x, k, r) \geq u_s((M_L(\hat{t}_L), M_R(\hat{t}_R)) \mid I^s, x, k, r) \tag{6}$$

assuming that a sender who is indifferent chooses to send a truthful message.

A.1.1 Two-players Communication

Lemma 1 *In a two-players cheap-talk game, truthful revelation through cheap talk is always possible if and only if preferences are not divergent with respect to the state variables and both voters belong to the same group of voters, regardless of the bias gap of the players.*

Step 1: *Voting decision under truthful communication.*
First I focus on voting decisions of the receiver when she believes the sender is communicating truthfully. It is trivial to analyse the case in which the sender is perfectly informed about the state of the world – i.e. the sender's information is $I^s = (m, m)$. If the sender sends a truthful pair of messages to her receiver, $(M_L(m), M_R(m))$. It follows that a left (right) biased receiver $r < 1/2$ ($r > 1/2$), regardless of the information she has obtained from parties' advertisement, will always vote for her favoured party's candidate. It follows that if the receiver is partially informed only about her favoured party's candidate, because she knows either through direct exposure to the parties' advertisement or through sender's truthful messages that her favoured party has a moderate candidate, then she will always vote for the party she likes most. Now suppose the receiver has no information about the candidate type of her favoured party. Suppose the receiver knows that the left party's candidate is a moderate and has no information about the right party's candidate, that is the information hold by the receiver is $I^r = (m, M_L(t_L), \emptyset, M_R(\emptyset))$ or $I^r = (t_L, M_L(m), \emptyset, M_R(\emptyset))$, or $I^r = (m, M_{1,L}(m), \emptyset, M_{1,R}(\emptyset))$. In terms of posterior beliefs for the receiver the three information set are equivalent, whenever she believes the sender's messages are truthful, thus

$$\rho(m, m \mid I^r, x, k) = \frac{\sigma_R(1 - x_R(m))^2}{\sigma_R(1 - x_R(m))^2 + (1 - \sigma_R)}$$

and the receiver will vote for the left party if and only if

$$u_r(1 \mid I^r, x, k) > u_r(0 \mid I^r, x, k)$$

which holds for any r such that

$$r < \frac{1}{2} + \frac{m}{4} \left[1 - \rho(m, m \mid I^r, x, k)\right].$$

The uninformed receiver, $I^r = (\emptyset, M_L(\emptyset), \emptyset, M_R(\emptyset))$ will obviously vote for her favoured party, i.e. will vote for the left party if and only if $u_r(1 \mid I^r, x, k) > u_r(0 \mid I^r, x, k)$ which holds for $r < 1/2$.

Step 2: *Incentive Compatibility.*
It is obvious that a sender has incentive to strategically transmit her information to the receiver she is matched with if and only if the information transmission can affect the outcome of the election through the voting decision of the receiver. If the sender can communicate strategically, truthful revelation is incentive compatible for the sender if and only if

$$u_s((M_L(t_L), M_R(t_R)) \mid I^s, x, k, r) \geq u_s((M_L(\hat{t}_L), M_R(\hat{t}_R)) \mid I^s, x, k, r).$$

From step 1 follows that whenever a receiver is informed about her favoured party's candidate, she will always cast her vote in his favour. Then, to simplify the argument let us assume the sender is matched with a receiver that belongs to the group of left independent voters, the receiver bliss point is such that $r \in (\frac{1}{2} - \frac{m}{4}\frac{1-\sigma_L}{\sigma_L(1-x_L)^{k+1}+1-\sigma_L}, \frac{1}{2})$. First, consider a perfectly informed sender, $I^s = (m, m)$. The sender's expected payoffs from every possible message pair are:

$$\begin{aligned}
u_s(M_L(m), M_R(m) \mid I^s, x, k, r) &= u_s((M_L(m), M_R(\emptyset)) \mid I^s, x, k, r) = m - s \\
u_s(M_L(\emptyset), M_R(m) \mid I^s, x, k, r) &= x_L(m)(m-s) + \\
&+ (1 - x_L(m))(-1 + m + s) \\
u_s(M_L(\emptyset), M_R(\emptyset) \mid I^s, x, k, r) &= (1 - x_R(m)(1 - x_L(m)))(m - s) + \\
&+ (x_R(m)(1 - x_L(m)))(-1 + m + s)
\end{aligned}$$

A perfectly informed sender sends a truthful messages pair to a left biased sender if and only if

$$u_s((M_L(m), M_R(m)) \mid I^s, x, k, r) \geq u_s(M_L(\emptyset), M_R(m) \mid I^s, x, k, r)$$

and

$$u_s((M_L(m), M_R(m)) \mid I^s, x, k, r) \geq u_s(M_L(\emptyset), M_R(\emptyset) \mid I^s, x, k, r)$$

that hold if and only if $s \leq 1/2$. In words, the sender will truthfully reveal her information if both voters in the matched pair are ex-ante biased toward the same party, otherwise the sender will send a babbling message.

Suppose the sender is informed about the receiver's less favoured candidate, that is her information is $I^s = (\emptyset, m)$.

$$
\begin{aligned}
u_s(M_L(\emptyset), M_R(m) \mid I^s, x, r) &= \rho(m, m \mid I^s, x, k)[x_L(m)(m-s) \\
&+ (1 - x_L(m))(-1 + m + s)] \\
&+ \rho(e, m \mid I^s, x, k)(-1 + m + s) \\
u_s(M_L(m), M_R(\emptyset) \mid I^s, x, r) &= u_s(M_L(m), M_R(m) \mid I^s, x, k) \\
&= \rho(m, m \mid I^s, x, k)(m - s) \\
&+ \rho(e, m \mid I^s, x, k)(m/2 - s)
\end{aligned}
$$

$$
\begin{aligned}
u_s(M_L(\emptyset), M_R(\emptyset) \mid I^s, x, r) &= \rho(m, m \mid I^s, x, k)[x_L(m)^k(m - s) \\
&+ ((1 - x_L(m))x_R(m))(-1 + m + s) \\
&+ (1 - x_R(m)(1 - x_L(m)))(m - s)] \\
&+ \rho(e, m \mid I^s, k, r)[(1 - x_R(m))(m/2 - s) \\
&+ x_R(m)(-1 + m + s)]
\end{aligned}
$$

From the previous case it is easy to see why a sender will never lie by revealing false information about the candidate's type of the right party. Suppose both players in the matched pair are ex-ante biased toward the same political party. Then if both are right biased reporting the truth will increase the probability that their favoured party wins the election; if they are both left biased the sender knows the receiver will trust her message and with probability $(1 - \sigma_L)$ will take the wrong decision of voting for an extremist candidate. It is obvious that if the sender is biased for the right party and the receiver is biased for the left party, then sending the message pair $(M_L(m), M_R(\emptyset))$ is never optimal from the point of view of the sender. The same argument applies also to the message pair $(M_L(m), M_R(m))$. Thus I am left with only two message pairs, $(M_L(\emptyset), M_R(m))$ and $(M_L(\emptyset), M_R(\emptyset))$, to analyse. Incentive compatibility implies that a sender with bliss point s will send a truthful message if and only if

$$
u_s(M_L(\emptyset), M_R(m) \mid I^s, x, k, r) \geq u_s(M_L(\emptyset), M_R(\emptyset) \mid I^s, x, k, r)
$$

and

$$u_s(M_L(\emptyset), M_R(m) \mid I^s, x, k, r) \geq u_s(M_L(m), M_R(\emptyset) \mid I^s, x, k, r)$$

After some manipulation, it is easy to infer that the sender will send the message pair $(M_L(\emptyset), M_R(m)$ if and only if

$$s \geq \frac{1}{2} - \frac{m}{4} \frac{1-\sigma_L}{\sigma_L(1-x_L)^2 + 1 - \sigma_L}$$

that is, whenever the sender is an ex-post independent voter.

The last case left to analyse is the message strategies of the uniformed sender, $I^s = (\emptyset, \emptyset)$. The expected payoffs of a sender matched with an ex-post left biased independent receiver, from each messages pair are:

$$\begin{aligned}
u_s(M_L(m), M_R(m) \mid I^s, x, k, r)) &= u_s((M_L(m), M_R(\emptyset)) \mid I^s, x, k, r) = \\
&= (m-s)(\rho_s(m, m \mid I^s, k) + \rho_s(m, e \mid I^s, k)) \\
&+ (m/2 - s)(\rho_s(e, m \mid I^s, k) + \rho_s(e, e \mid I^s, k))
\end{aligned}$$

$$\begin{aligned}
u_s(M_L(\emptyset), M_R(m) \mid I^s, x, k, r) &= \rho_s(m, m \mid I^s, k)(x_L(m)(m-s) \\
&+ (1 - x_L(m))(-1 + m + s)) \\
&+ \rho_s(m, e \mid I^s, k)(x_L(m)(m-s) \\
&+ (1 - x_L(m))(-1 + m/2 + s)) \\
&+ \rho_s(e, m \mid I^s, k)(-1 + m + s) \\
&+ \rho_s(e, e \mid I^s, k)(-1 + m/2 + s) \\
u_s(M_L(\emptyset), M_R(\emptyset) \mid I^s, x, k, r) &= \rho_s(m, m \mid I^s, k)(m-s)\Big((1 - x_R(m)(1 - x_L(m))) \\
&+ x_R(m)^k (1 - x_L(m))(-1 + m + s)\Big) \\
&+ \rho_s(e, m \mid I^s, k)(x_R(m)(-1 + m + s) \\
&+ (1 - x_R(m))^k (m/2 - s)) + \rho_s(m, e \mid I^s, k)(m-s) \\
&+ \rho_s(e, e \mid I^s, k)(m/2 - s)
\end{aligned}$$

It is immediate to see that an uniformed sender will prefer to send a truthful message to a receiver biased in opposite direction rather than lie in favour of the receiver's favoured party:

$$u_s(M_L(m), M_R(m) \mid I^s, x, k, r) < u_s(M_L(\emptyset), M_R(\emptyset) \mid I^s, x, k, r)$$

for any $s > 1/2$. But, an uninformed sender may want to lie about the candidate's type of her favoured party of the less favoured receiver's party, i.e.

$$u_s(M_L(m), M_R(\emptyset) \mid I^s, x, k, r) < u_s(M_L(\emptyset), M_R(\emptyset) \mid I^s, x, k, r)$$

and after some manipulation it is easy to see that a sender prefers to lie about the receiver's favoured party's candidate, i.e.

$$u_s(M_L(m), M_R(\emptyset) \mid I^s, x, k, r) > u_s(M_L(\emptyset), M_R(\emptyset) \mid I^s, x, k, r)$$

whenever

$$s \leq \frac{1}{2} - \frac{m}{4} \frac{1-\sigma_L}{\sigma_L(1-x_L)^2 + 1 - \sigma_L}$$

Thus, an independent uniformed sender, in equilibrium, will send truthful message pair to her receiver if and only if she is biased toward the favoured party of her receiver and she belongs to the group of ex-post independent voters. In other words, if a sender is matched with a left biased ex-post independent receiver she sends a truthful message pair whenever

$$\frac{1}{2} - \frac{m}{4} \frac{1-\sigma_L}{\sigma_L(1-x_L)^2 + 1 - \sigma_L} \leq s \leq \frac{1}{2}$$

and a babbling message otherwise. Whenever both the sender and the receiver are both ex-post independent, that is their preferences are not divergent with respect to the state variables, and are ex-ante biased towards the same party, the sender will prefer to send a truthful message pair to her receiver, otherwise, i.e. if the sender and the receiver have divergent political preferences or they are ex-ante biased toward different parties, the sender will send a babbling message pair.

It is trivial to check that, from sequential rationality and consistency requirements, it follows that, whenever the receiver rationally updates her beliefs, the sender matched with a receiver with ex-post divergent political preferences will always send a babbling messages' pair, and the receiver will disregard all the messages received from a sender with ex-post divergent political bias.

A.1.2 The k-voters Communication Game

Let restrict the attention to the equilibrium strategies of the two-players communication game. This is without loss of generality, given that within the sampled network of each voter there is no preference uncertainty and the communication among voters has the form of private messages. Thus each voter to evaluate the messages she receives one by one and she can disregard all message pairs received from the senders biased for the party she likes less. Denote by $\beta \in (0,1)$ the probability that in each of the k matched pair both voters are ex-ante biased toward the same party.

In order to show that an ex-post independent sender never deviates from the message strategies described in the two-players game even when the communication network is richer, it is sufficient to show that there is not profitable deviation. Suppose $(k-1)$ senders follow the equilibrium strategies of the two-player cheap talk game described before, and let analyse the incentive to deviate of the k^{th} sender, with information I^s.

Theorem 1. *In any pairwise k-players cheap-talk game with restricted state space there are two ideological cutoffs that depend on the network's richness k and the degree of homophily β, given the equilibrium level of advertisement of the two parties. The two cutoofs display the configuration $q_l < 1/2 < q_r$ such that*

$$q_l = \frac{1}{2} - \frac{m}{4}\frac{1-\sigma_L}{1-\sigma_L+\sigma_L(1-x_L(m))^{\beta k+1}},$$

and

$$q_r = \frac{1}{2} + \frac{m}{4}\frac{1-\sigma_R}{1-\sigma_R+\sigma_R(1-x_R(m))^{\beta k+1}}.$$

Truthful revelation through cheap talk is possible if and only if the bliss points of both voters in each matched pair belong to the interval defined by those cutoffs and voters' ex-ante biases have same direction. There does not exist any other configuration of the biases of voters such that truthful information transmission is possible.

Consider again the case in which the sender is matched with a left biased independent receiver.

First, consider a perfectly informed sender, $I^s = (m,m)$, the sender's expected payoffs from every messages pair are:

$$\begin{aligned}
u_s(M_L(m), M_R(m) \mid I^s, x, k, \beta) &= u_s((M_L(m), M_R(\emptyset)) \mid I^s, x, k, \beta) \\
&= m - s \\
u_s(M_L(\emptyset), M_R(m) \mid I^s, x, k, \beta) &= x_L(m)^{\beta(k-1)+1}(m-s) \\
&+ (1 - x_L(m))^{\beta(k-1)+1}(-1+m+s) \\
u_s(M_L(\emptyset), M_R(\emptyset) \mid I^s, x, k, \beta) &= [(1 - x_R(m))(1 - x_L(m))]^{\beta(k-1)+1}(m-s) \\
&+ [(x_R(m)(1 - x_L(m)))]^{\beta(k-1)+1}(-1+m+s)
\end{aligned}$$

and a perfectly informed sender sends a truthful message to a left biased receiver if and only if

$$u_s((M_L(m), M_R(m)) \mid I^s, x, k, \beta) \geq u_s(M_L(\emptyset), M_R(m) \mid I^s, x, k, \beta)$$

and

$$u_s((M_L(m), M_R(m)) \mid I^s, x, k, \beta) \geq u_s(M_L(\emptyset), M_R(\emptyset) \mid I^s, x, k, \beta)$$

that holds if and only if $s > 1/2$, i.e. voters in the matched pair are biased toward the same party, then the sender has no incentive to deviate from the strategy prescribed in the two-players game when she is perfectly informed.

Now consider the case in which the sender is informed about the receiver's less favoured candidate, that is her information set is $I^s = (\emptyset, m)$. To simplify the notation let $\hat{\rho}(t_L, t_R) = \rho(t_L, t_R \mid I^s, k, \beta)$, and $\gamma = \beta(k-1)+1$, denote the belief of the receiver. The sender's expected payoffs from each messages pair, when a left biased receiver believes she is behaving informatively, are:

$$\begin{aligned}
u_s(M_L(\emptyset), M_R(m) \mid I^s, x, k, \beta) &= \hat{\rho}(m,m)[x_L(m)^\gamma(m-s) \\
&+ (1 - x_L(m))^\gamma(-1+m+s)] \\
&+ (1 - \hat{\rho}(m,m))(-1+m+s) \\
u_s(M_L(\emptyset), M_R(\emptyset) \mid I^s, x, k, \beta) &= \hat{\rho}(m,m)[x_L(m)^\gamma(m-s) \\
&+ ((1 - x_L(m))x_R(m))^\gamma(-1+m+s) \\
&+ ((1 - x_R(m))(1 - x_L(m))^\gamma(m/2 - s)] \\
&+ (1 - \hat{\rho}(m,m))[x_R(m)^\gamma(-1+m+s) \\
&+ (1 - x_R(m))^\gamma(m/2 - s)]
\end{aligned}$$

And from incentive compatibility, a sender with bliss point s will lie to her receiver if and only if

$$u_s(M_L(\emptyset), M_R(m) \mid I^s, x, k, \beta) < u_s(M_L(\emptyset), M_R(\emptyset) \mid I^s, x, k, \beta).$$

Thus, the sender will send a truthful message pair, $(M_L(\emptyset), M_R(m))$ if and only if

$$s \geq \frac{1}{2} - \frac{m}{4} \frac{1-\sigma_L}{\sigma_L(1-x_L)^{\beta k+1} + 1 - \sigma_L}.$$

It follows that the sender never deviates from the equilibrium strategy prescribed in the two-player cheap talk game when her information is $I^s = (\emptyset, m)$.

It is left to analyse the case in which the sender is uniformed, $I^s = (\emptyset, \emptyset)$. The expected payoffs of a sender matched with a left biased independent receiver, from each messages pair are:

$$\begin{aligned}
u_s(M_L(m), M_R(m) \mid I^s, x, k, \beta) &= u_s((M_L(m), M_R(\emptyset)) \mid I^s, x, \theta) \\
&= (\hat{\rho}(m,m) + \hat{\rho}(m,e))(m-s) \\
&+ (\hat{\rho}(e,m) + \hat{\rho}(e,e))(m/2-s) \\
\\
u_s(M_L(\emptyset), M_R(m) \mid I^s, x, k, \beta) &= \hat{\rho}(m,m)(x_L(m)^\gamma(m-s) \\
&+ (1-x_L(m))^\gamma(-1+m+s)) \\
&+ \hat{\rho}(m,e)(x_L(m)^\gamma(m-s) \\
&+ (1-x_L(m))^\gamma(-1+m/2+s)) \\
&+ \hat{\rho}(e,m)(-1+m+s) \\
&+ \hat{\rho}(e,e)(-1+m/2+s) \\
u_s(M_L(\emptyset), M_R(\emptyset) \mid I^s, x, k, \beta) &= \hat{\rho}(m,m)\Big((((1-x_L(m)) \\
&\quad (1-x_R(m)))^\gamma + x_L(m)^\gamma)(m-s) \\
&+ (x_R(m)(1-x_L(m)))^\gamma(-1+m+s)\Big) \\
&+ \hat{\rho}(e,m)(x_R(m)^\gamma(-1+m+s) \\
&+ (1-x_R(m))^\gamma(m/2-s)) \\
&+ \hat{\rho}(e,e)(m/2-s) + \hat{\rho}(m,e)(m-s).
\end{aligned}$$

It is immediate to see that an uniformed sender will prefer to send a truthful message to a receiver biased in opposite direction rather than lie in favour of the receiver's favoured party:

$$u_s(M_L(m), M_R(m) \mid I^s, x, k, \beta) < u_s(M_L(\emptyset), M_R(\emptyset) \mid I^s, x, k, \beta)$$

for any $s > 1/2$. But, an uninformed sender may want to lie about the candidate's type of the receiver's favoured party, i.e.

$$u_s(M_L(m), M_R(\emptyset) \mid I^s, x, k, \beta) > u_s(M_L(\emptyset), M_R(\emptyset) \mid I^s, x, k, \beta)$$

and after some manipulation it is easy to see that a sender prefers to lie about the receiver's favoured party's candidate whenever

$$s < \frac{1}{2} - \frac{m}{4} \frac{1 - \sigma_L}{\sigma_L (1 - x_L)^2 + 1 - \sigma_L}.$$

Thus, an uniformed sender will send truthful messages pair if and only if she belongs to the group of ex-post independent and she is biased toward the favoured party of her receiver, i.e. if a sender is matched with a left biased ex-post independent receiver she sends a truthful message pair whenever

$$\frac{1}{2} - \frac{m}{4} \frac{1 - \sigma_L}{\sigma_L (1 - x_L)^2 + 1 - \sigma_L} \leq s \leq \frac{1}{2}$$

and a babbling message otherwise.

In other words, whenever a sender is an ex-post independent voter she has no incentive to deviate from the equilibrium strategies of the two-players communication game.

A.2 Advertising Strategies

I first derive the optimal advertising level for the cost c when parties do not target their advertisement, then I show how homophily and network richness affect the targeting choices of the parties. Then I compare the maximum unit cost a party is willing to pay to advertise its candidate when the targeting technology is not available, i.e. party can only randomly advertise their candidates to both groups of independent voters, with the case in which a group-targeting technology is available, and how those choices are affected by both the homophily, β, and the richness of the network's structure, k.

Proof of Theorem 2 *In a symmetric pure strategy equilibrium parties never tailor their advertisements exclusively to their own supporters, and there exist critical levels of richness of the network, k, and degree of homophily, β, such that party L advertises a moderate candidate exclusively to the opponent's supporters if and only if the cost of advertisement is such that*

$$c^{**}(k, \beta) < \frac{(2-3m)(1-\sigma_R)}{4},$$

it randomly advertises a moderate candidate if and only if

$$c^{*}(k, \beta) < \frac{(1-\sigma_R)(\beta k+1)\rho(m, e \mid I, \beta, k)(2-3m)}{16}$$

and never advertise a moderate candidate otherwise.

Before proceeding with the analysis remember that information never travel between the two groups of independent voters, regardless of their bias gap (Theorem 1).

A.2.1 Random Advertising

Suppose that both parties randomly advertise their moderate candidates to the two groups of independent voters. Consider the case in which the left party, L, has a moderate candidate, $t_L = m$, and assume it advertises him, $x_L(m) > 0$. Note that if both parties have a moderate candidate, given they play in symmetric strategies, then the left party wins the election with probability $\pi(m, m \mid x) = 1/2$. If the right party has instead an extremist candidate what matter is the distribution of indifferent voters for each information they hold. If the candidate's type of the left party is a moderate the information that a voter may hold are $I^{\beta,k} = \{(m, M_L(m), \emptyset, M_R(\emptyset)), (\emptyset, M_L(m), \emptyset, M_R(\emptyset)),$ $(m, M_L(\emptyset), \emptyset, M_R(\emptyset)), (\emptyset, M_L(\emptyset), \emptyset, M_R(\emptyset))\}$. The indifferent voter for each information is

$$i(m, M_L(m), \emptyset, M_R(\emptyset)) = \frac{1}{2} + \frac{m}{4}\rho(m, e \mid I, x, k, \beta)$$

where

$$\rho(m, e \mid I, x, k, \beta) = \frac{1-\sigma_R}{1-\sigma_R(m)+\sigma_R(m)(1-x_R)^{\beta k+1}}$$

moreover notice

$$i(m, M_L(m), \emptyset, M_R(\emptyset)) = i(m, M_L(\emptyset), \emptyset, M_R(\emptyset)) = i(\emptyset, M_L(m), \emptyset, M_R(\emptyset))$$

given that voters are able to screen among sources of information and they take into account only valuable information, and

$$i(\emptyset, M_L(\emptyset), \emptyset, M_R(\emptyset)) = \frac{1}{2}.$$

It follows that

$$\mu(x \mid \theta, k, \beta) = \frac{1}{2} + \frac{m}{4}\rho(m, e \mid I, \beta, k)(1 - (1 - x_L(m))^{\beta k+1})$$

and $\mu(x \mid \theta, k, \beta) \in (1/2 - m/4, 1/2 + m/4)$ which implies that

$$\pi(m, e \mid x) = \frac{4 + \rho(m, e \mid I, x, k, \beta)(1 - (1 - x_L(m))^{\beta k+1})}{8}$$

i.e. $\pi(m, e \mid x, k, \beta) \in (0, 1)$, for all $x_L(m) \in (0, 1)$. Party L's expected utility from advertising a moderate candidate is

$$\begin{aligned} U_L(x_L(m) \mid \theta, k, \beta) &= \sigma_R \Big[\frac{1}{2}(1 - 2m) - \frac{2 - 3m}{2}\Big] \\ &+ (1 - \sigma_R) \Big[4 + \frac{2 - 3m}{16}\rho(m, e \mid I, x, k, \beta) \\ &\quad (1 - (1 - x_L(m))^{\beta k+1}) - (1 - m)\Big] - c x_L(m) \end{aligned}$$

and the optimal level of advertisement $x_L^*(m) \in (0, 1)$ solves

$$(1 - \sigma_R(m))(\beta k + 1)\rho(m, e \mid I, \beta, k)(1 - x_L(m))^{\beta k} = \frac{16c}{2 - 3m}. \tag{7}$$

Then, if $(1 - \sigma_R(m))(\beta k + 1)\rho(m, e \mid I, x, k, \beta)(2 - 3m)/16 < \hat{c}$ party L will never randomly advertise a moderate candidate and $x_L(m) = 0$, otherwise it will advertise a moderate if and only if the expected utility form advertising a moderate candidate with intensity $x_R^*(m)$, $U_L^{\emptyset}(x_L^* \mid \theta, k, \beta)$, is higher than the expected utility of not advertise it, $U_L^{\emptyset}(\tilde{x} \mid \theta, k, \beta)$, where $\tilde{x}_L(m) = 0$. The expected payoff of the left party when she does observe that its candidate is a moderate, and decides to not advertise is

$$\begin{aligned} U_L^{\emptyset}(\tilde{x} \mid \theta, k, \beta) &= \sigma_R(m)\Big[\Big(\frac{1}{2} - \frac{\rho(e, m \mid I, \beta, k)(1 - (1 - x_R(m)^{(\beta k+1)})}{8}\Big) \\ &\quad (1 - 2m) - \frac{2 - 3m}{2}\Big] + (1 - \sigma_R(m))\Big[\frac{1}{2}\frac{2 - 3m}{2} + (1 - m)\Big] \end{aligned}$$

Thus $U_L(x_L(m)|\theta,k,\beta) \geq U_L^{\varnothing}(\tilde{x} \mid \theta,k,\beta)$ if and only if

$$cx_L(m) \leq \frac{\rho(e \mid I,\beta,k)(2-3m+\sigma_R(m)m)(1-(1-x_L(m))^{(\beta k+1)}}{16} \tag{8}$$

Denote by $p = 1 - x_L(m)$ and $\sigma_J(m) = \sigma_J$, then using the equation (4) in the equation (8), after some manipulation we get

$$\tilde{c} \leq \frac{\rho(e \mid I,\beta,k)(2-3m+\sigma_R m)(2-3m)(1-\sigma_R)(\beta k+1)}{16[p(2-3m+\sigma_R m)+(1-p)(2-3m)(1-\sigma_R)(\beta k+1)]} \tag{9}$$

Putting together these observations, it follows that for any value of $p \in (0,1)$, $\tilde{c} \geq \hat{c}$ where the optimal level of advertisement, $x_L^*(m)$ solves

$$(1-\sigma_R(m))(\beta k+1)\rho(m,e \mid I,\beta,k)(1-x_L(m))^{\beta k} = \frac{16c}{2-3m},$$

and party randomly advertises a moderate candidate whenever

$$c^*(k,\beta) \leq \frac{(1-\sigma_R(m))(\beta k+1)\rho(m,e \mid I,\beta,k)(2-3m)}{16}$$

and never advertises a moderate candidate otherwise.

A.2.2 Targeting Advertisement

Now consider the case in which parties can decide whether and to which group they can target the advertisement. Assume a party can either decide to randomly advertise her candidate or target its advertisement to one of the two groups of independent voters. Consider the strategy x^* in which parties always randomly advertise a moderate candidate for $c < c^*(k,\beta)$ to both groups. Suppose the candidate's type of party L is a moderate and the L party can deviate by tailoring its advertising to only one of the two groups of independent voters with intensity $y_L^j = x_L^*(m)$, where $j = \{l,r\}$ denotes toward which party the targeted group of voters is biased ex-ante (for example y_L^l denotes the choices of the left party to advertise its candidate excursively to the group of ex-ante left biased independent voters). I assume that when the advertisement is targeted to only one group all the individuals that belong to such group are perfectly informed. Let assume, in what follows, that the right party has no access to the targeting technology.

First, suppose the left party targets its own supporters. Given that all left biased independent voters are informed about the candidate's type of their favoured party, party L wins the electoral competition with probability $1/2$, regardless if its opponent has a moderate or an extremist candidate. If party L, instead, randomly advertises its candidate it wins the electoral competition with probability $\pi_L(m, t_R \mid y_L^l, x_R) = 1/2$ if the opponent's candidate is moderate, and with higher probability, $\pi_L(m, e \mid x) = 1/2 + \rho(m, e \mid I, \beta, k)(1 - (1 - x_L(m))^{(\beta k+1)} > 1/2$, if the opponent's candidate is an extremist. Thus, if a party has a moderate candidate it always prefer to advertise him to both groups rather than target its own supporters.

Now suppose party L targets its advertisement to the opponent's supporters. If both parties have a moderate candidate, then party L, by tailoring the advertisement to the opponent's supporters leaves all its own supporters uninformed, and some of them will vote for the opponent whenever they are (directly or indirectly) informed about the party R's candidate (the fraction of left biased independent voters who votes for the right party decreases as the homophily and the richness of the network decrease). Moreover, given that the party R randomly advertises its candidate also some right biased independent voters will be (directly or indirectly) informed and vote for the right party. Thus, if both parties have a moderate candidate and party L targets the opponent supporters, then the left party wins the electoral competition with probability $\pi_L(m, m \mid y_L^r, x_R) = 1/2 - (\rho(e \mid I, \beta, k)(1 - 2(1 - x_R(m)))/16 < 1/2$, that is by randomly advertising its candidate party L will increase the probability of winning the election if both candidate are moderate. Conversely, when the opponent's candidate is an extremist, by targeting the opponent's supporters, party L wins the election with probability $\pi_L^r(m, e \mid y) = 1$.

Then the left party randomly advertises its moderate candidate if and only if $U_L(x_L(m) \mid \theta, \beta, k) \geq U_L(y_L^r(m) \mid \theta, \beta, k)$ that gives us

$$\begin{aligned}(1 \quad - \quad \sigma_R)&\Big[\Big(\frac{\rho(e \mid I, \beta, k)(1 - (1 - x_R)^{\beta k+1})}{8} - \frac{1}{2}\Big)\Big(\frac{2 - 3m}{2}\Big)\Big] \\ + \quad \sigma_R&\Big(\frac{\rho(e \mid I, \beta, k)(1 - 2(1 - x_R)^{\beta k+1}}{8}\Big)(1 - 2m) > 0,\end{aligned}$$

which never holds for any $x_R(m) \in (0, 1)$. Thus whenever a technology that allows targeting different groups of independent voters is available a party that observes a moderate candidate will prefer to target the supporters of its opponent.

Now I prove that in symmetric strategies both party advertise a moderate candidate and target the supporter of their opponent rather than do not advertise a moderate candidate. The party L does advertise a moderate candidate and targets the opponent's supporter if and only if $U_L(y_L^r(m) \mid \theta, \beta, k) \geq U_L(x_L(m) \mid \theta, \beta, k)$:

$$\sigma_R\left[\frac{1}{2}(1-2m) - \frac{2-3m}{2}\right] + (1-\sigma_R)\left[\frac{2-3m}{2} - (1-m)\right] - cy^r \geq$$
$$\sigma_R\left[\frac{1}{2}(1-2m) - \frac{2-3m}{2}\right] + (1-\sigma_R)\left[\frac{1}{2}\frac{2-3m}{2} - (1-m)\right]$$

that implies

$$cy_L^r < \frac{(2-3m)(1-\sigma_R)}{4}$$

thus, whenever the cost of targeting the advertisement, c^τ, is such that $c^\tau > \frac{(2-3m)(1-\sigma_R)}{4}$, $y_L^r = 0$ and the left party does not target the advertisement to the opponent's supporters. Concluding, if the network structure is such that

$$\frac{4}{\beta k + 1} < \rho(m, e \mid I, \beta, k)$$

parties randomly advertise a moderate candidate if and only if the cost of advertisement, $c(k, \beta)$, is such that

$$c^*(k, \beta) < \frac{(1-\sigma_R)(\beta k + 1)\rho(m, e \mid I, \beta, k)(2-3m)}{16},$$

and never advertise otherwise. On the other hand, if the degree of homophily and/or the richness of the network are low enough, i.e.

$$\frac{4}{\beta k + 1} > \rho(m, e \mid I, \beta, k)$$

then parties randomly advertise a moderate candidate if and only if

$$\frac{(1-\sigma_R)(\beta k + 1)\rho(m, e \mid I, \beta, k)(2-3m)}{16} < c^*(k, \beta) < \frac{(2-3m)(1-\sigma_R)}{4}$$

and they prefer to target their advertisement to the opponent's supporter if and only if

$$c^*(k, \beta) < \frac{(2-3m)(1-\sigma_R)}{4}.$$

Parties never tailor their advertisement to their own supporters.

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