

# Collusive Agreements in Vertically Differentiated Markets

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June 2016<sup>2</sup>.

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<sup>2</sup>This survey is forthcoming in L. C. Corchon, M. A. Marini, (eds.) *Handbook of Game Theory and Industrial Organization*, Cheltenham, Edward Elgar, 2017. I am very grateful for valuable discussions and suggestions to Rabah Amir, Maria Rosa Battaglion, Luca Benvenuti, Sergio Currarini, Alberto De Santis, Jean Gabszewicz, Michael Kopel, Sébastien Mitraillet, Michele Polo, Giorgio Rodano, Ornella Tarola and the participants to seminars at the University of Milan, Universidad Carlos III of Madrid, Nova Universidade de Lisboa, Université Paris Dauphine and Università di Rome La Sapienza. I gratefully acknowledge the financial support from CIRIEC International, Liège, Belgium.

## Abstract

This survey introduces a number of game-theoretic tools to model collusive agreements among firms in vertically differentiated markets. I firstly review some classical literature on collusion between two firms producing goods of exogenous different qualities. I then extend the analysis to a  $n$ -firm vertically differentiated market to study the incentive to form either a whole market alliance or partial alliances made of subsets of consecutive firms in order to collude in prices. Within this framework I explore the price behaviour of groups of colluding firms and their incentive to either pruning or proliferating their products. It is shown that a *selective* pruning within the cartel always occurs. Moreover, by associating a partition function game to the  $n$ -firm vertically differentiated market, it can be shown that a sufficient condition for the *cooperative* (or coalitional) stability of the whole industry cartel is the equidistance of firms' products along the quality spectrum. Without this property, and in presence of large quality differences, collusive agreements easily loose their stability. In addition, introducing a standard infinitely repeated-game approach, I show that an increase in the number of firms in the market may have contradictory effects on the incentive of firms to collude: it can make collusion *easier* for bottom and intermediate firms and *harder* for the top quality firm. Finally, by means of a three-firm example, I consider the case in which alliances can set endogenously qualities, prices and number of variants on sale. I show that, in every formed coalition, (i) *market pruning* dominates *product proliferation* and (ii) partial cartelisation always arises in equilibrium, with the bottom quality firm always belonging to the alliance

**Keywords:** *Vertically differentiated market, price collusion, product pruning, product proliferation, endogenous qualities, endogenous alliance formation, coalition structures, grand coalition, coalition stability, core, simultaneous and sequential game of coalition formation.*

**JEL Classification:** D42, D43, L1, L12, L13, L41.

# 1 Introduction

This survey primarily focusses on the incentives of firms to sign collusive agreements in vertically differentiated markets as, for instance, in cartels, mergers and alliances. It also studies the effects of collusion on market prices and qualities.

The relationship between mergers and price-quality combinations has recently attracted increasing attention in empirical and theoretical I. O. literature.<sup>1</sup> On the empirical ground, Berry and Waldfogel (2001) found for instance a negative correlation between merging operations and number of existing radio stations with, in addition, an observed increase in radio formats varieties related to mergers. Sweeting (2010) and George (2007) reported similar evidence for U.S. radio music industry and Fan (2013) for U.S. newspapers market. In airline industries, Peters (2006) observed a reduction of flight frequency in those market segments in which merging carriers compete most, while Mazzeo (2003) showed a deterioration of on-time performances following airline mergers.

In this chapter we introduce a number of game-theoretic tools that can be used to model firm collusive agreements in vertically differentiated markets. Section 2 quickly reviews the initial literature on price collusion in a vertically differentiated duopoly. Section 3 introduces a vertically differentiated oligopoly setting to study in more detail the incentives of firms to form either the whole market cartel or partial cartels made of a subsets of adjacent firms in the product space, with the aim to collude in prices. This exercise allows us to characterize the price behaviour of alliances by looking, in particular, at the behaviour of what we denote, in turn, *bottom*, *intermediate* and *top* cartels, with this meaning arbitrary cartels made of adjacent firms and including either the bottom or the top quality firm (in the *bottom* and *top* cartel, respectively) or made of intermediate firms only (in the *intermediate* cartel). It can be shown that at the price equilibrium for any top or intermediate cartel only two variants remain on sale from the cartel, the highest and the lowest quality good produced by the cartel. On the other hand, in any bottom cartel, only one variant remains on sale, namely the highest quality among those produced *ex ante* by the cartel. The remaining sections focus on the stability of collusion. Section 4, by associating a partition function game to the  $n$ -firm vertically differentiated market shows as a sufficient condition for the coalitional stability of the whole industry cartel is the *equidistance* of all firms' qualities. Without this feature, and in presence of highly asymmetric quality gaps, collusive agreements may be easily become

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<sup>1</sup>Among the others, Mazzeo 2002, Crawford and Shum 2006, Gandhi *et al.* 2008, Draganska *et al.* 2009, Chu 2010, Byrne 2012, Fan 2013, Lee 2013.

unstable. Section 5 introduces a standard infinite horizon game to show that an increase in the number of firms in the market may have contradictory effects on the incentive of firms to collude: collusion may become *easier* for bottom and intermediate firms and *harder* for the top quality firm. Finally, in Section 6, by means of a three-firm example, I consider the case in which colluding firms can also decide endogenously their quality and price combinations. In such case, once merged, firms are allowed to optimally reshape their qualities and prices according to the new market structure. From this, it can be checked whether full or partial cartelisations can be sustained as a subgame perfect equilibria of the whole game, which now includes a coalition formation process taking place at the first stage. For this model we show that partial cartelisation always arises in equilibrium with the bottom quality firm always belonging to the formed cartel. Section 7 concludes.

## 2 Collusion in a Vertically Differentiated Duopoly

In his seminal paper Hackner (1994) analyses the relationship between collusion and vertical product differentiation in an infinitely repeated duopoly framework. The main issue here is to see whether price collusion is more or less likely to be sustained when the quality gap between firms' products is higher. It is shown that the monopoly pricing is more easily reachable when products are closer along the quality ladder. Also, among the two firms, the top quality firm is the one possessing the highest incentive to break a collusive agreement. This is because with a large quality gap the profit of the top quality firm is high even without collusion, and this makes the incentive to collude for this firm weaker than for the bottom quality firm. In a related paper, Ecchia and Lambertini (1997) study how the stability of price collusion in a vertically differentiated duopoly can be affected by the introduction of a minimum quality standard. The presence of a welfare-maximizing minimum quality standard can make the full collusive agreement harder to sustain. This is because the quality standard decreases the product differentiation providing the bottom quality firm with a stronger temptation to defect.<sup>2</sup>

from the above analyses, two things can be noticed. The first is that, in both models considered above, the degree of product differentiation does not change after a coalition has formed, since the collusive behavior is restricted to pricing. This assumption is a natural entry point in the literature on cartel stability under product differentiation, as it enables to disentangle the effect of quality gap on the stability of cartels. Further, conceiving collusion

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<sup>2</sup>The contradictory results among the two papers mainly depends on their different cost assumptions.

in terms of pricing is particularly reasonable from a short-run perspective. Still, it leaves unexplored a companion question, namely the effect of the cartel on product differentiation. This analysis could be particularly pregnant in a long-run perspective since one cannot exclude that in a more extended time span a coalition (typically a cartel or a merger) entails structural changes, such as relocations of production facilities, or adjustment in the product range and quality.

The second is, instead, that in both papers the market is duopolistic and, as a result, any cooperation between the two firms implies by definition a full market cartelisation. There exist remarkable examples in which firms form partial alliances (i.e. those including a subset of firms in the market) rather than the whole market coalition. Actually, In partial alliances colluding firms can still compete against rival firms outside the coalition, and the effects of partial alliances or mergers are not equivalent to those observed when all firms mimic the behaviour of a monopolist.

Lambertini (2000) explores how cartel stability can be connected to the R&D activity in a duopoly in which the collusive quality choice may occur either under price or quantity-setting behaviour.<sup>3</sup> The issue concerning the alliance formation with more than two firms in a vertically differentiated market remain, however, unexplored, as also the effect of partial collusion on market equilibrium. Scarpa (1998) models a vertical differentiation market with three firms competing in quality and prices.<sup>4</sup> In particular, he considers the role of a minimum quality standard, and highlights how the demand level of each firm in a vertically differentiated market only depends on quality and price of adjacent firms in the product space. Indeed, since only adjacent variants compete against each other, under partial collusion defining the optimal set of products to market requires to put in balance *the cannibalization effect* that a variant produced by the coalition exerts *within* the coalition with the possibility that this variant *steals* consumers from the rival firms (*stealing effect*).

Other related papers are those by Lommerud and Sorgard (1997), Gandhi *et al.* (2008), Chen and Schwartz (2013) and Brekke *et al.* (2014), all devoted to the analysis of price-quality post-merger re-positioning.<sup>5</sup> The first paper is inspired by Salant *et al.* (1983) and Deneckere and Davidson (1985) and it is devoted to evaluate the profitability of a merger under both

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<sup>3</sup>A different strand of literature considers the possible impacts of R&D joint ventures on product market collusion. See on this, Martin (1995), Lambertini *et al.* (2002) and Marini *et al.* (2014).

<sup>4</sup>recently Pezzino (2010) develops the same model under quantity competition. Cesi (2010) studies the effect of two-firm mergers in a three-firm market in presence of a social-welfare maximizing minimum quality standard.

<sup>5</sup>Other recent papers by Mazzeo (2002), Einav (2003) and Seim (2006) look at the price-quality strategies decided by industry entrants.

Cournot and Bertrand competition. The authors assume that the market is initially populated by three firms and, therefore, two firms can merge and decide on the number of brands to market. When the fixed cost of marketing a brand is ‘high’, the merged entity reduces its product range. This increases the profitability of mergers both under Bertrand and Cournot competition due to reduced marketing costs. With a ‘low’ cost of marketing, the effect on the product range depends both on the nature of competition and on the degree of product differentiation. For example, under Cournot or Bertrand competition and sufficiently differentiated products, the non-merging firm finds profitable to introduce a new brand, thereby damaging the merged entity. In order to highlight the impact of a merger on non-price competition, Gandhi *et al.* (2008) assume instead that firms can instantaneously and costlessly reposition their products after a merger, thereby choosing both price and location in a Hotelling market. They show that after a merger the products are repositioned away from each other to reduce the resulting cannibalization effect. Consequently, non-merging substitutes are repositioned between the merged products and, after all these location strategies, the merged firm’s incentive to raise prices decreases. Similarly, in a Hotelling framework, Chen and Schwartz (2013) analyse the incentive for firms to introduce a product innovation when proposing a merger-to-monopoly. In contrast to Arrow’s finding for process innovation, where the monopolist never undertakes R&D efforts to innovate, in this paper the incentive to invest in incremental product innovations can be higher for the merged entity (a monopolist) than for a rival facing competition from the existing good. Indeed, the monopolist can coordinate the pricing of the two products overcompensating the erosion of profits coming from cannibalization. In a spatial competition model *à la* Salop with three *ex ante* identical firms, Brekke *et al.* (2014) show that any two-firm merger reduces its product quality whereas the non-merging firm responds increasing its quality. Final prices can either increase or decrease according to the responsiveness of demand functions. Moreover, it is shown that if a merger entails the closure of one of the two merged firms, this always leads to higher qualities and prices for all firms in the market.

### **3 Collusion in a $N$ -firm Vertically Differentiated Market**

As underlined above, although easily interpretable, a two-firm vertically differentiated market possesses a few limitations and does not allow a full-fledged analysis of market partial cartelisation. Therefore, in this first modelling section we simply extend a traditional model *à la* Mussa and Rosen (1978) and Gabszewicz and Thisse (1979) to a  $n$ -firm market in order to see

the main implications in terms of pricing behaviour under collusion.<sup>6</sup>

Let  $n$  firms  $k = 1, 2, \dots, n$  supply  $n$  different quality variants  $q_1, q_2, \dots, q_n$  with  $q_k \in (0, \infty)$  and  $q_n > q_{n-1} > \dots > q_1$  to a population of consumers. As in Mussa and Rosen (1978) and Gabszewicz and Thisse (1979) consumers are indexed by  $\theta$  and uniformly distributed in the interval  $[0, \beta]$ , with  $\beta < \infty$ . As usual, the parameter  $\theta$  captures consumers' willingness to pay for quality: the higher  $\theta$ , the higher the baseline utility gained when consuming variant  $q_k$  of the product. Each consumer can either buy one unit of a variant or not buying at all. Formally, a simple way to represent consumer's utility is

$$U(\theta) = \begin{cases} \theta q_k - p_k & \text{when buying variant } k \\ 0 & \text{when not buying.} \end{cases} \quad (1)$$

where  $p_k$  is the price set by firm  $k$ , such that  $p_k \in [0, \bar{p}]$ , where  $0 < \bar{p} < \infty$  is a given upper bound on prices. From the above formulation, the marginal consumer buying variant  $k = 1$  is

$$\theta_1 = \frac{p_1}{q_1},$$

and the market is partially *uncovered*, with some consumers excluded from buying even the bottom-quality variant. In general, the consumer indifferent between buying variant  $k - 1$  and  $k$  for  $k = 2, 3, \dots, n$  is

$$\theta_k = \frac{p_k - p_{k-1}}{q_k - q_{k-1}}.$$

where  $p_k > p_{k-1}$  for every  $k = 1, 2, 3, \dots, n$ . For the time being, we assume that product qualities are exogenously given and we disregard costs to simplify calculations.<sup>7</sup>

When considering price competition, the payoffs of all firms can be easily characterized by describing the payoff of three types of firms in the quality spectrum: (i) top quality, (ii) intermediate quality and (iii) bottom quality firm. The top quality firm (denoted  $k = n$ ) sets a price  $p_n$  to maximize its profit

$$\Pi_n = D_n p_n = \left( \beta - \frac{p_n - p_{n-1}}{q_n - q_{n-1}} \right) p_n. \quad (2)$$

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<sup>6</sup>In their seminal paper Gabszewicz and Thisse (1980) introduce a  $n$ -firm model of vertically differentiated firms under the assumption of *equispaced* products.

<sup>7</sup>The existence of quality fixed costs does not alter the nature of the results obtained here.

Conversely, every intermediate firm  $k = 2, 3, \dots, n - 1$  maximizes

$$\Pi_k = D_k p_k = \left( \frac{p_{k+1} - p_k}{q_{k+1} - q_k} - \frac{p_k - p_{k-1}}{q_k - q_{k-1}} \right) p_k. \quad (3)$$

Finally, the bottom-quality firm ( $k = 1$ ), maximizes

$$\Pi_1 = D_1 p_1 = \left( \frac{p_2 - p_1}{q_2 - q_1} - \frac{p_1}{q_1} \right) p_1. \quad (4)$$

The optimal reply of every noncooperative firm can be easily obtained as follows:

$$p_n(p_{n-1}) = \frac{1}{2} (p_{n-1} + \beta(q_n - q_{n-1})). \quad (5)$$

for the *top quality* firm ( $k = n$ )

$$p_k(p_{k-1}, p_{k+1}) = \frac{1}{2} \frac{p_{k-1}(q_{k+1} - q_k) + p_{k+1}(q_k - q_{k-1})}{(q_{k+1} - q_{k-1})}. \quad (6)$$

for every *intermediate quality* firm  $k = 2, 3, \dots, n - 1$  and

$$p_1(p_2) = \frac{1}{2} \frac{p_2 q_1}{q_2} \quad (7)$$

for the *bottom quality* firm ( $k = 1$ ).

Expressions (2)-(4) show that prices and qualities are strategic complements for all firms ( $\frac{\partial^2 \Pi_k}{\partial p_k \partial q_k} > 0$ ) and the best-reply of every firm shifts outward as due to an increase in its quality. On the other hand, for every firm  $k$ , an increase in the quality of direct rivals' products  $q_j$ , for  $j = (k + 1)$  and  $(k - 1)$  causes a negative effect on its profit ( $\frac{\partial \Pi_k}{\partial p_k \partial q_j} < 0$ ) and price-competition becomes tougher as a result. Note also that, from (2)-(4), all firms' profit functions are concave in their own prices and also their choice sets are compact and convex and their best-replies are *contractions*,<sup>8</sup> in such a way that the existence of a unique (noncooperative) Nash equilibrium  $n$ -price vector  $p^*$  associated to the  $n$  variants  $(q_1, q_2, \dots, q_n)$  is guaranteed for any (finite) number of firms competing in the market.<sup>9</sup>

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<sup>8</sup>See Gabszewicz *et. al* (2016a).

<sup>9</sup>See, for instance Friedman (1991), p.84.



### 3.1 Full price collusion

When firms form the whole market cartel, they can be assumed to maximize the sum of all firms' payoffs:

$$\Pi_{\{N\}} = \sum_{k=1}^n \Pi_k = \Pi_1 + \dots + \Pi_{k-1} + \Pi_k + \Pi_{k+1} + \dots + \Pi_n.$$

For every colluding firm  $k = 1, \dots, n$ , the first-order condition writes as<sup>10</sup>

$$\frac{\partial \Pi_{\{N\}}}{\partial p_k} = \frac{\partial \Pi_{k-1}}{\partial p_k} + \frac{\partial \Pi_k}{\partial p_k} + \frac{\partial \Pi_{k+1}}{\partial p_k} = 0. \quad (8)$$

Since the top quality-firm  $k = n$  in the cartel internalizes the payoff of its lower-quality neighbour, its optimal reply writes as

$$p_n^c(p_{n-1}) = p_{n-1} + \frac{\beta}{2} (q_n - q_{n-1}). \quad (9)$$

Along the same rationale, for all intermediate firms  $k = 2, 3, \dots, (n-1)$  which are members of the cartel, the optimal reply writes as

$$p_k^c(p_{k-1}, p_{k+1}) = \frac{p_{k-1}(q_{k+1} - q_k) + p_{k+1}(q_k - q_{k-1})}{(q_{k+1} - q_{k-1})}, \quad (10)$$

since they internalize the payoff of their adjacent neighbour members of the cartel. Finally, the optimal reply of the bottom quality firm  $k = 1$  is given by

$$p_1^c(p_2) = \frac{q_1}{q_2} p_2. \quad (11)$$

As already pointed out by Gabszewiz *et al.* (1986) and, more recently, by Gabszewicz *et al.* (2017), in a model in which unit costs vary only mildly with quality, under full price collusion the  $n$  firms set prices  $p_k^c$  such that their market shares are nil for all firms except for the top-quality one ( $k = n$ ). In particular, under full collusion, for every firm  $k = 1, 2, \dots, n$  and  $j < k$ , it is easy to obtain prices as

$$p_k^c = \frac{1}{2} \beta \sum_{j=1}^k (q_j - q_{j-1}). \quad (12)$$

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<sup>10</sup>Note that  $\frac{\partial^2 \Pi_N}{\partial p_i^2} = -\frac{2(v_{i+1} - v_{i-1})}{(v_{i+1} - v_i)(v_i - v_{i-1})} < 0$  for  $i = 2, 3, \dots, n-1$ , and, therefore, the joint profit  $\Pi_N$  is concave in every firm's price  $p_i$ . The same condition holds for the two extreme firms along the quality spectrum, i.e.  $i = 1$  and  $i = n$ .

Inserting (12) in every firm's market share  $D_k$ , we obtain for the bottom quality firm,

$$D_1(p_1^c, p_2^c) = \left( \frac{p_2^c - p_1^c}{\tau_2} - \frac{p_1^c}{\tau_1} \right) = \left( \frac{\frac{1}{2}\beta(\tau_1 + \tau_2) - \frac{1}{2}\beta\tau_1}{\tau_2} - \frac{\frac{1}{2}\beta\tau_1}{\tau_1} \right) = 0$$

where  $\tau_j = (q_j - q_{j-1})$  denotes the quality gap of every firm  $j$  selling goods of lower or equal quality than firm  $k$ , and  $\tau_1 = (q_1 - q_0) = q_1$ . Moreover, inserting (12) in every *intermediate quality* firm's market share  $\tau_k$ , we obtain:

$$\begin{aligned} D_k(p_{k-1}^c, p_k^c, p_{k+1}^c) &= \left( \frac{p_{k+1}^c - p_k^c}{\tau_{k+1}} - \frac{p_k^c - p_{k-1}^c}{\tau_{k-1}} \right) = \\ &= \left( \frac{\frac{1}{2}\beta \sum_{j \leq k+1} \tau_j - \frac{1}{2}\beta \sum_{j \leq k} \tau_j}{\tau_{k+1}} - \frac{\frac{1}{2}\beta \sum_{j \leq k} \tau_j - \frac{1}{2}\beta \sum_{j \leq k-1} \tau_j}{\tau_{k-1}} \right) = \\ &= \left( \frac{\frac{1}{2}\beta\tau_{k+1}}{\tau_{k+1}} - \frac{\frac{1}{2}\beta\tau_k}{\tau_k} \right) = 0. \end{aligned}$$

with,

$$\begin{aligned} D_n(p_{n-1}^c, p_n^c) &= \left( \beta - \frac{p_n^c - p_{n-1}^c}{q_n - q_{n-1}} \right) = \left( \beta - \frac{\frac{1}{2}\beta \sum_{j \leq n} \tau_j - \frac{1}{2}\beta \sum_{j \leq n-1} \tau_j}{\tau_n} \right) = \\ &= \left( \beta - \frac{\frac{1}{2}\beta\tau_n}{\tau_n} \right) = \frac{1}{2}\beta, \end{aligned}$$

for the *top quality* firm. Thus, when colluding together all firms cover only half of the market and the whole market payoff is:

$$\Pi_{\{N\}} = \sum_{k \in N} \Pi_k^{\{N\}} = \sum_{k=1}^n p_k^c D_k = \frac{1}{4}\beta^2\tau_n. \quad (13)$$

### 3.2 Partial cartels

In many cases firms can organize themselves in a coalition structure (partition) of the  $N$  firms different from the grand coalition,  $C = (S_1, S_2, \dots, S_m)$ , with  $m \leq n$ . However, in a vertically differentiated market every firm can effectively distort prices by colluding either with its left (lower quality) or with right (higher quality) or with both its local competitors.<sup>11</sup> In what follows we introduce a few simple definitions helping to develop the analysis of partial cartelisation. In order to affect prices, firms can form *bottom*, *intermediate* or *top* quality cartels. To each of these members, the first order condition of profit maximization writes as:

<sup>11</sup>Price collusion can also occur among disconnected firms, but in this case the prices of the firms will just be equal to those arising at the noncooperative equilibrium.

(i) in the case of *interior* cartel members:

$$\frac{\partial \Pi_S}{\partial p_k} = \frac{\partial \sum_{k \in S} \Pi_k}{\partial p_k} = \frac{\partial \Pi_{k-1}}{\partial p_k} + \frac{\partial \Pi_k}{\partial p_k} + \frac{\partial \Pi_{k+1}}{\partial p_k} = 0,$$

leading to the optimal reply function

$$p_k^{pc}(p_{k-1}, p_{k+1}) = \frac{p_{k-1}(q_{k+1} - q_k) + p_{k+1}(q_k - q_{k-1})}{(q_{k+1} - q_{k-1})}, \quad (14)$$

where the superscript *pc* stands for *partial collusion*.

(ii) In the case of *lower boundary* cartel member:

$$\frac{\partial \Pi_S}{\partial p_k} = \frac{\partial \sum_{k \in S} \Pi_k}{\partial p_k} = \frac{\partial \Pi_k}{\partial p_k} + \frac{\partial \Pi_{k+1}}{\partial p_k} = 0,$$

leading to the best-reply function

$$p_k^{pc}(p_{k-1}, p_{k+1}) = \frac{\frac{1}{2}p_{k-1}(q_{k+1} - q_k) + p_{k+1}(q_k - q_{k-1})}{(q_{k+1} - q_{k-1})}. \quad (15)$$

(iii) Finally, in the case of *upper boundary* cartel member:

$$\frac{\partial \Pi_S}{\partial p_k} = \frac{\partial \sum_{k \in S} \Pi_k}{\partial p_k} = \frac{\partial \Pi_k}{\partial p_k} + \frac{\partial \Pi_{k-1}}{\partial p_k} = 0,$$

leading to the best-reply function

$$p_k^{pc}(p_{k-1}, p_{k+1}) = \frac{p_{k-1}(q_{k+1} - q_k) + \frac{1}{2}p_{k+1}(q_k - q_{k-1})}{(q_{k+1} - q_{k-1})}. \quad (16)$$

**Definition 1** (i) A *bottom cartel*  $S_B \subset N$  is a coalition formed by consecutive intermediate firms  $k = 2, \dots, n-1$ , also including the bottom quality firm  $k = 1$ . (ii) An *intermediate cartel*  $S_k \subset N$  is a coalition formed by at least two consecutive intermediate firms  $k = 2, \dots, n-1$ . (iii) A *top cartel*  $S_T \subset N$  is a coalition formed by consecutive intermediate firms  $k = 2, \dots, n-1$ , also including the top quality firm  $k = n$ .

Following Gabszewicz *et al.* (2016a), the next proposition characterizes the market shares of firms belonging to a: (i) intermediate cartel; (ii) bottom cartel; (iii) top cartel.

**Proposition 1** (i) A *bottom cartel* only produces in equilibrium the top quality variant among those in the cartel. (ii) Any *intermediate cartel* only produces in equilibrium the top and the

bottom quality variants among those in cartel. (iii) Any top cartel only produces in equilibrium the top and the bottom quality variants among those in the cartel.

**Proof.** See the Appendix. ■

**Corollary 1** *In a generic partition of the  $n$  firms  $P = (S_1, S_2, \dots, S_m)$  organized in  $m < n$  non trivial cartels, a total of  $2m + (n - z) - 1$  (resp.  $2m + (n - z)$ ) variants are put on sale in the market when the partition includes (resp. does not include) the bottom cartel, for  $z = s_1 + s_2 + \dots + s_m$ , where  $s_j$ , for  $j = 1, 2, \dots, m$ , denotes the cardinality of every cartel.*

In order to complete the characterization of every partial cartelisation of the market we can provide a price comparison for all firms under partial cartelisation with respect to both fully noncooperative and fully collusive cases.

**Proposition 2** *Under partial cartelisation the firms set prices  $p_k^{pc}$  higher or equal than the corresponding prices  $p_k^*$  charged at the noncooperative price equilibrium and lower than the corresponding full collusive prices  $p_k^c$ .*

**Proof.** Let us assume, for simplicity, that only one cartel  $S \subset N$  has formed, and that the remaining firms play as singletons. Note, however, that the same reasoning would apply to the case with more than one cartel. It can be easily checked that the joint profit of an arbitrary cartel  $\Pi_S$  is continuous and concave with respect to every firm's price  $p_k$ , for  $k \in S$ . Moreover, the optimal reply of every partially collusive firm  $k \in S$  is a *contraction* and, hence, a unique partially collusive price profile  $p^{pc}$  exists for any given level of qualities  $q_1, q_2, \dots, q_n$ , under the assumption that all firms with unsold goods set their equilibrium prices exactly at the levels for which the sales become nil.<sup>12</sup> Thus, we can: (a) start with a profile  $p^*$  of Nash equilibrium prices. (b) Let the firms in  $S \subset N$  reply according to their optimal collusive replies. A quick comparison between optimal replies under partial cartelisation (14)-(16) and purely noncooperative Nash equilibrium shows that the former are always steeper than the latter and, since they are in both cases positively sloped, all firms in the cartel will set higher prices than in the noncooperative scenario. (c) Similarly an increase in prices will also occur to all firms in the fringe playing noncooperatively: given the higher prices of the cartel, they will respond, in turn, increasing their prices. (d) The described adjustment process, given the contraction property of all firms' optimal replies, will converge to a new profile of prices such that  $p_k^{pc} > p_k^*$  for all  $k = 1, 2, \dots, n$ . Inequality  $p_k^c > p_k^{pc}$ , for every  $k = 1, 2, \dots, n$ , can be proved along similar lines. ■

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<sup>12</sup>Any higher price would be equally optimal since these goods are not purchased by consumers.

## 4 A Cooperative Approach to the Stability of the Whole Industry Agreement

In this section we consider the incentive of firms to form the whole industry cartel (grand coalition). Following Gabszewicz et al (2016), a partition function game can be associated to the vertically differentiated market introduced in Section 3 and, from this, it can be proved that the core of this game is nonempty when the qualities of the products sold by the firms are equispaced along the quality spectrum. Moreover, it can be easily shown that, when this regularity condition does not hold, the core can be empty. Therefore, a fully collusive agreement among firms is more easily reachable when there are neither too large nor too asymmetric gaps between firms' qualities. The symmetry in quality gaps helps to maintain the discipline of the whole market cartel just because it reduces the incentive of firms to free-ride by leaving the agreement.

Following Gabszewicz et al. (2016) we adopt the concept of *delta core* by Hart and Kurz (1983), also denoted *projection core* in the recent axiomatization by Bloch and van den Nouweland (2014). Since for the case of vertically differentiated markets the coalitional worth possesses positive coalition externalities,<sup>13</sup> the delta or projection-core is the smallest core and, therefore, its existence implies the existence of all other possible versions of core in games with simultaneous moves.<sup>14</sup>

To prove our result we can formally associate to the vertically differentiated market described above a *partition function game*  $P = (N, v(S, C))$ , where  $N$  is the set of firms and  $v(S, C) \in \mathcal{R}_+$  is the worth associated to every coalition of firms  $S \subset N$  belonging to a coalition structure  $C \in \mathcal{C}$ . In our model, when a cartel  $S \subset N$  forms, its maximal coalitional payoff is obtained when the remaining firms in  $N \setminus S$  stick together in the complementary coalition  $\{N \setminus S\}$ . Therefore, if the core of the partition function game  $P$  exists when the coalitional worth  $v(S, C)$  is computed for  $C = (S, N \setminus S)$ , it will *a fortiori* exist for any other partition of the firms in  $N \setminus S$ .

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<sup>13</sup>This means that every firm is advantaged when rivals merge in coalitions

<sup>14</sup>Gabszewicz *et al* (2016b) use this notion of core in order to provide the strongest core existence result. Demanges (1994) provides general conditions for core existence in economies producing differentiated goods, although in absence of externalities between coalitions. Zhao (2013) examined the existence of  $\alpha$ -,  $\gamma$ - and  $\delta$ -core in a three-firm linear Cournot oligopoly with different marginal costs. In a differentiated quantity oligopoly with three (or four firms) Watanabe and Matsubayashi (2013) show that for any degree of product differentiation the  $\gamma$ -core is nonempty while the  $\delta$ -core only exists in presence of high product differentiation. For a more detailed account of the works dealing with coalitional agreements in oligopoly games, see Marini (2009) and Currarini and Marini (2015).

**Definition 2** The core of partition function game  $P = (N, v(S, C))$  consists of all efficient payoff allocations  $\Pi \in \mathcal{R}_+^{|N|}$  respecting  $\sum_{k \in S} \Pi_k \geq v(S, C)$  for all  $S \subset N$  and for all partition  $C \in \mathcal{C}$  to which  $S$  may belong to.

Then we have the following result:

**Proposition 3** Let market variants  $q_1, q_2, \dots, q_n$  be equispaced with  $(q_k - q_{k-1}) = \tau \in (0, \infty)$  for all  $k = 1, 2, \dots, n$ , with  $q_0 = 0$ . Then, the core of the partition function game  $P = (N, v(S, C))$  associated to the  $n$ -firm vertically differentiated market is nonempty.

The proof of this result, contained in Gabszewicz *et al.* (2016), is constructive and it finds a specific allocation of the monopoly profit  $\Pi^{\{N\}}$  such that neither an individual firm nor a bottom, intermediate or top cartel have an incentive to leave the grand coalition under its maximal expectation, i.e. that the remaining firms continue to collude inside the complementary cartel  $N \setminus S$ . Such allocation is simply

$$\Pi = \left( s_1 \Pi^{\{N\}}, s_2 \Pi^{\{N\}}, \dots, s_n \Pi^{\{N\}} \right) \quad (17)$$

where  $s_k$  for  $k = 1, 2, \dots, n$  are shares of the monopoly profit given by

$$s_k = \frac{\Pi_{\{k\}}^{\{k, N \setminus k\}}}{\sum_{k \in N} \Pi_{\{k\}}^{\{k, N \setminus k\}}},$$

such that  $\sum_{k \in N} s_k = 1$ , where  $\Pi_{\{k\}}^{\{k, N \setminus k\}}$  is the profit of every firm  $k$  when in competition with its complementary coalition  $N \setminus \{k\}$ .

As the simple example below shows, when the quality gaps among firms widely differ, the core can be empty.

#### 4.1 An Empty Core Example

Let us assume four firms in the market, i.e.  $N = \{1, 2, 3, 4\}$ , initially selling four different qualities  $q_1, q_2, q_3, q_4$ . Let now the firms fully collude by forming the grand coalition. Let now the top cartel  $S_T = \{2, 3, 4\}$  decide to leave the grand coalition and coalition structure  $C = (\{1\}, \{2, 3, 4\})$  form as a result. In this case, the top cartel obtains:

$$\Pi_T^{\{1\}, \{2, 3, 4\}} = \frac{\beta^2 q_2 q_3 (q_3 - q_2)}{(4q_3 - q_2)^2} + \frac{1}{4} \frac{\beta^2 (4q_2 q_4 - q_1 q_4 - 3q_1 q_2)}{(4q_2 - q_1)}.$$

For  $\beta = 1$ ,  $q_1 = 1$ ,  $q_2 = 5$  and  $q_4 = 10$  and  $q_3 > 7.26$ , the quality gap between  $q_2$  and  $q_3$  (both produced inside the cartel) becomes sufficiently high for

$$\Pi_T^{\{\{1\},\{2,3,4\}\}} + \Pi_1^{\{\{1\},\{2,3,4\}\}} > \Pi_N^{\{N\}} = \frac{1}{4}\beta^2 q_4 = 2.5$$

and the core is, therefore, empty. If all products are equipaced, with  $q_1 = 2.5$ ,  $q_2 = 5$ ,  $q_3 = 7.5$  and  $q_4 = 10$ ,

$$\Pi_1^{\{\{1\},\{2,3,4\}\}} + \Pi_T^{\{\{1\},\{2,3,4\}\}} = 2.21 < \Pi_N^{\{N\}}.$$

Moreover, it can be checked that all other feasible deviations by single or coalitions of firms do not improve upon the grand coalition allocations. Core existence is, in such a way, guaranteed.

## 5 A Noncooperative Approach to the Stability of the Whole Industry Agreement

In this section we test the stability of the whole industry cartel using a standard repeated-game approach. For this purpose, we use the model with equipaced variants, which is sufficiently tractable.<sup>15</sup>

We already obtained in Section 3 the monopoly payoff. What is required to characterize the standard *grim* strategy of a standard infinite-horizon extension of the vertically differentiated model is to make explicit the noncooperative equilibrium payoffs of all firms and, as a second aspect, to define their *defection* payoffs obtained when playing their best-replies when all other rivals collude. Finally, an intuitive *allocation rule* has to be introduced to divide the fully collusive payoff among the  $n$  heterogeneous firms. In what follows we derive the price vector obtained at the Nash equilibrium of every *constituent* game under the equipaced product assumption.

**Proposition 4** *Let market variants  $q_1, q_2, \dots, q_n$  be equipaced and such that  $q_k - q_{k-1} = \tau$  for every  $k = 1, 2, \dots, n$ , with  $q_0 = 0$ . Then, the noncooperative Nash equilibrium price vector for all firms  $k = 1, 2, \dots, n$  is given by:*

$$p_k^* = \frac{\tau\beta (b_1^k - b_2^k)}{\sqrt{3}b_1^n + \sqrt{3}b_2^n},$$

and for  $b_1 = (2 + \sqrt{3})$  and  $b_2 = (2 - \sqrt{3})$ .

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<sup>15</sup>In this section we use part of the material contained in Bos and Marini (2016).

**Proof.** See the Appendix. ■

If we assume the existence of quadratic quality costs for each firm  $c(q_k) = \frac{q_k^2}{2}$ , their noncooperative payoffs can be written as

$$\Pi_k^* = \left( \frac{p_{k+1}^* - p_k^*}{\tau} - \frac{p_k^* - p_{k-1}^*}{\tau} \right) p_k^* - \frac{q_k^2}{2} = \frac{2}{3} \frac{\tau \beta^2 (b_1^k - b_2^k)^2}{(b_1^n + b_2^n)^2} - \frac{(\tau k)^2}{2}.$$

Now, since the fully collusive price under equally spaced variants is, for every firm  $k = 1, 2, \dots, n$

$$p_k^c = \frac{1}{2} \beta \tau k,$$

we can easily characterize the fully collusive profit of every firm (before any transfer takes place) as

$$\Pi_k^c = \frac{\beta^2 \tau (k)}{4} - \frac{(\tau k)^2}{2}.$$

One way to devide the fully collusive profit among all firms is to use the following natural quality-ranking:

$$r_k = k \cdot \tau$$

for every  $k = 1, 2, \dots, n$ , which substantially corresponds to the position of each firm in the equispaced quality space. Therefore, using the fact that

$$\sum_{k=1, \dots, n} r_k = \frac{n(n+1)\tau}{2}$$

at the fully collusive agreement we can simply assign to every firm a personalised share equal to:

$$\alpha_k = \frac{r_k}{\sum_{k=1, \dots, n} r_k} = \frac{2k}{n(n+1)}.$$

Finally, using every firm's noncooperative best-replies we can easily obtain every firm's defection profit as:

$$\begin{aligned} \Pi_1^d &= \left( \frac{p_2^c - 2p_1^d}{\tau} \right) p_1^d - \frac{\tau^2}{2} = \left( \frac{\frac{1}{2}\beta\tau(2) - 2(\frac{1}{4}\tau\beta)}{\tau} \right) \frac{1}{4}\tau\beta - \frac{\tau^2}{2} = \frac{1}{8}\tau\beta^2 - \frac{\tau^2}{2}, \\ \Pi_k^d &= \left( \frac{p_{k+1}^c - 2p_k^d + p_{k-1}^c}{\tau} \right) p_k^d - \frac{(\tau(k))^2}{2} = \frac{(k)^2\tau\beta^2}{8} - \frac{(\tau(k))^2}{2} \\ \Pi_n^d &= \left( \beta - \frac{p_n^d - p_{n-1}^c}{\tau} \right) p_n^d - \frac{(\tau n)^2}{2} = \frac{(n+1)^2\tau\beta^2}{16} - \frac{(\tau n)^2}{2}. \end{aligned}$$



Thus, for the full collusion to be sustained as a subgame perfect Nash equilibrium of the infinite horizon game (via a grim strategy) the discount factor of every firm has to respect the following condition

$$\delta_k(\beta, \tau, n) \geq \frac{\Pi_k^d - \alpha_k \Pi_k^c}{\Pi_k^d - \Pi_k^*} = \frac{\left(\frac{(k)^2}{8} \tau \beta^2 - \frac{(\delta(k))^2}{2}\right) - \frac{2(k)}{n(n+1)} \left(\frac{\beta^2 \tau n}{4} - \frac{(\tau n)^2}{2}\right)}{\frac{(k)^2}{8} \beta^2 \tau - \frac{2}{3} \frac{\beta^2 \tau \left((b_1)^k - (b_2)^k\right)^2}{((b_1)^n + (b_2)^n)^2}} \quad (18)$$

for all  $k = 1, 2, \dots, n - 1$  and

$$\delta_n(\beta, \tau, n) \geq \frac{\Pi_n^d - \alpha_n \Pi_n^c}{\Pi_n^d - \Pi_n^*} = \frac{\left(\frac{n^2 \beta^2 \tau}{8} - \frac{(\tau n)^2}{2}\right) - \frac{2n}{n(n+1)} \left(\frac{\beta^2 \tau n}{4} - \frac{(\tau n)^2}{2}\right)}{\frac{(n+1)^2}{16} \tau \beta^2 - \frac{2}{3} \frac{\tau \beta^2 \left((b_1)^n - (b_2)^n\right)^2}{((b_1)^n + (b_2)^n)^2}} \quad (19)$$

for  $k = n$ .<sup>16</sup> From the above expressions, the following proposition follows.

**Proposition 5** *Let market variants  $q_1, q_2, \dots, q_n$  be equispaced with  $q_k - q_{k-1} = \tau$  for every  $k = 1, 2, \dots, n$  and  $q_0 = 0$ . Let also every firm's share of the monopoly profit be determined by its quality ranking, as  $\alpha_k = 2(k)/n(n+1)$ . Then, an increase in the number of firms  $n$  reduces the discount factor sustaining the fully cooperative agreement as a subgame perfect Nash equilibrium of the infinitely repeated game via a grim strategy for all firms  $k = 1, 2, 3, \dots, n - 1$ , while it increases the discount factor of the top quality firm  $k = n$  (for  $n > 3$ ).*

**Proof.** This can be obtained from straightforward manipulations of expressions (10)-(11).

■

Proposition 5 helps to see that, in vertically differentiated markets, under equispaced variants, an increase in the number of firms has contradictory effects on the incentive of firms to collude: it makes collusion *easier* to sustain for bottom and for intermediate quality firms but, at the same time, it makes it harder for the top quality firm. This result is somehow surprising if compared to the usual view that collusion is more easily sustainable when the number of firms is small, whereas it becomes usually harder when the number of firms increases.

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<sup>16</sup>Note that a constraint for  $\beta > \sqrt{n\delta}\sqrt{2}$  must be imposed for both collusive and noncooperative firms' payoffs to be nonnegative.

## 6 Mergers and Alliances with Endogenous Qualities

To the best of our knowledge, a full-fledged theoretical study of the effects of alliances and mergers on market prices and qualities in a *vertically* differentiated industry with more than two firms has not yet been provided. Similarly unexplored is the analysis of *mergers stability* between firms in vertically differentiated markets when firms can re-shape prices and qualities of the products after mergers. Anecdotal evidence shows that mergers and acquisitions often occur among firms selling fairly differentiated products along the quality spectrum. For instance, some of the mergers taking place after 1979 deregulation of U.S. airline market, occurred between one big national/international carrier and one low fare local carrier (e.g. the merger between American Airlines and AirCal in 1986 or between Delta and Atlantic Southeast Airlines in 1999)<sup>17</sup> or, alternatively, just between intermediate-quality carriers (as for Southwest Airlines and AirTran Airways in 2010 or between Republic Airways and Midwest Airlines in 2009). Analogously, the European Airlines industry has witnessed a high number of mergers occurring between broadly differentiated airlines as, for instance, between Air France and Air-Inter in 1999 or between Lufthansa and Air Dolomiti in 2003.<sup>18</sup>

In a similar vein, the automotive industry is plenty of examples of premium car producers taking over economy car manufacturers, as in the merger occurring between Volkswagen Group and Skoda in 1991 or between Nissan and Renault in 1999.

One consequence of these consolidation processes is often which to re-position the lower quality brand towards a higher segment of the market or, in some other cases, to *un-brand* intermediate quality products as to create a *fighting brand* able to compete more aggressively with the firms positioned at the bottom of the quality spectrum. However, the latter strategy appears usually more as a temporary than a permanent strategy, since a fighting brand may risk to cannibalize the market of the merging firms. Ultimately, a consolidated group can find more advantageous to *re-brand* its economy products rather than *un-brand* some of its intermediate quality outlets. Instead of letting Smart for Two competing in the low segment of the market, Daimler-Benz preferred to transform this city car into a premium car. Similarly, the boom of mergers recently observed in pharmaceutical industries, involving top pharmaceutical companies acquiring generics drugs manufacturers (as in the recent case of Teva absorbing Allergan Generics), could have represented a similar trend.<sup>19</sup>

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<sup>17</sup>See: <http://www.airlines.org>.

<sup>18</sup>In some other cases the low-cost airlines have attempted to take over small-medium airlines, as in the recent hostile takeover launched by Ryanair to Air Lingus.

<sup>19</sup>See, for instance, Wieczner (2015),

In Gabszewicz *et al.* (2015) we introduce a simple framework in which three *ex ante* heterogeneous firms, initially producing three vertically differentiated goods, *low* (firm 1), *medium* (firm 2) and *high* (firm 3), enter a negotiation to decide whether to merge or not with some or all rival firms and, once merged, optimally reshape the qualities and prices according to the new market structure.

Assume as in Gabszewicz *et al.* (2015) a three-stage game where, at the first stage, every firm expresses its willingness to form an alliance or, alternatively, to stay as singleton. Then, at the second and third stage each formed coalition can decide, in turn, the qualities and prices of its goods. An alliance can either contains all firms in the market (*grand coalition*), as  $N = \{1, 2, 3\}$  or, alternatively, be formed by a nonempty subset  $S \subset N$  of firms, with  $S \in \mathcal{N}$ , where  $\mathcal{N} = 2^N \setminus \{\emptyset\}$  is the set of all nonempty coalitions of the  $N$  firms:

$$\mathcal{N} = (\{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}).$$

Thus, the set  $\mathcal{C}$  of all coalition structures  $C$  which can be formed by the three firms is:

$$\mathcal{C} = ((\{1\}, \{2\}, \{3\}), (\{1, 2\}, \{3\}), (\{1\}, \{2, 3\}), (\{1, 3\}, \{2\}), (\{1, 2, 3\})).$$

The game can be solved backward to analyse the prices and qualities selected in equilibrium by firms under the assumption that either the grand coalition or any other intermediate coalition structure has formed at the first stage. As in Bloch (1995, 1996) and Ray and Vohra (1999), the coalition formation game can be assumed *sequential*, with an exogenous order of play. Differently from them, since firms are *ex ante* heterogeneous, it is assumed that every firm proposes not only an alliance, but also a division of the coalition payoff. Each recipient of the proposal can either accept or reject the offer and, in case of rejection, it becomes its turn to make a proposal. The game is assumed finite-horizon and every firm only possesses one turn of proposal at each period.<sup>20</sup>

Since prices and qualities are selected in a sequence by every formed coalition, the payoffs accruing to a firm or a coalition in each feasible coalition structure can be easily obtained as follows:

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<sup>20</sup>Both Bloch's (1995, 1996) and Ray and Vohra's (1999) model a infinite-horizon negotiation process.

$(\{1\}, \{2\}, \{3\})$	$\Pi_1^* = 0.00005\beta^4$	$\Pi_2^* = 0.00124\beta^4$	$\Pi_3^* = 0.02348\beta^4$
$(\{1, 2, 3\})$	$\Pi_{\{1,2,3\}}^{\{N\}} = 0.03125\beta^4$		
$(\{1, 2\}, \{3\})$	$\Pi_{\{1,2\}}^{\{1,2\},\{3\}} = 0.00152\beta^4$	$\Pi_3^{\{1,2\},\{3\}} = 0.02443\beta^4$	
$(\{1, 3\}, \{2\})$	$\Pi_{\{1,3\}}^{\{1,3\},\{2\}} = 0.02443\beta^4$	$\Pi_2^{\{1,3\},\{2\}} = 0.00152\beta^4$	
$(\{1\}, \{2, 3\})$	$\Pi_1^{\{1\},\{2,3\}} = 0.00152\beta^4$	$\Pi_{\{2,3\}}^{\{1\},\{2,3\}} = 0.02443\beta^4$	

Table 1 - Firm payoffs in every coalition structure.

It turns out that, although the qualities and prices arising in each partial merger do not vary, the profits accruing to firms depend on the coalitions to (against) which they belong (compete).

Moreover, using the above payoffs, it can be shown that, although the full monopolization of the market is the most profitable outcome of the game, in a finite-horizon sequential game of coalition formation the incentive for firms to enter a whole industry merger is dominated by that to form partial mergers. In particular, the finite-horizon sequential coalition formation game reaches the results described by the following proposition.

**Proposition 6** (i) *When the high quality firm 3 is the initiator of the sequential coalition formation game, the only stable coalition structure is  $C_{12,3} = (\{1, 2\}, \{3\})$ , where firm 3 continues to produce the top variant  $q_3$  and the two remaining firms 1 and 2 only market intermediate variant  $q_2$ .* (ii) *When firm 2 is the initiator of the game, the only stable coalition structure is  $C_{13,2} = (\{1, 3\}, \{2\})$ , where firm 1 and 3 jointly produce top variant  $q_3$  and firm 2 produces intermediate variant  $q_2$ .* (iii) *Finally, when firm 1 is the initiator of the game, the only stable coalition structure is  $C_{12,3} = (\{1, 2\}, \{3\})$ , where firm 3 produces top variant  $q_3$  and 1 and 2 jointly produce intermediate variant  $q_2$ .*

**Proof.** See Gabszewicz et al. (2015). ■

Notice that, both in (i) and (ii) the initiator of the game never belongs to an alliance in equilibrium. Indeed, the payoff of a firm when it remains singleton (rationally expecting that the other firms will prefer to merge) dominates that of being part of the grand coalition, since in the latter case the distribution of profits would be unfavourable to the initial proposer. The equilibrium profit accruing to either firm 2 or 3 when initiating the game and competing against an alliance is, therefore, larger than when they are part of the alliance. The optimal strategy is, therefore, to induce the remaining firms to merge. A different result arises when firm 1 (the bottom quality one) begins the negotiation process. In this case, firm 1 cannot

credibly commit to remain independent since the remaining firms (2 and 3) prefer to play as singletons than forming an alliance (see Table 1). This is due to the fact that the alliance between firm 2 and 3 is problematic since in this circumstance 2 would optimally leapfrog the bottom quality firm, ending up by sharing the top quality firm's duopoly payoff, which is lower than the sum of firms' profits under triopoly. Knowing in advance the infeasibility of coalition  $\{2, 3\}$ , firm 1 would prefer to let firm 3 playing independently and, then, form an alliance with firm 2.

A striking result of this model is that all equilibrium mergers always contains the bottom quality firm which, in all cases, drops its low-quality variant from the market. In particular, whoever is the additional player included in a coalition (either the intermediate or the top quality firm), equilibrium prices and qualities always coincide with that observed in the case of a duopoly, with a high-quality firm competing against a low-quality rival, as in Motta (1992).

At first sight, this result seems to be counterintuitive. A natural conjecture would be that either the range of variants or the quality gap between variants in the market would change with the players involved in the alliance. This model shows instead that only profits accruing to the single firms change with the type of partial merger, range of products, quality gap and price being unchanged. Indeed, the cannibalization effect and the stealing effect induce the merger, whatever its members, to withdraw from the market the lowest quality variant between the set which can be produced *a priori*. Interestingly, depending on the intensity of these effects, in some circumstances this variant is withdrawn from the market at the price stage, in some other circumstances at the quality stage. In particular, the merger formed by the intermediate quality and by the low-quality firm stops immediately to market the bottom-quality product at the price stage. In contrast, the merger formed by the top and the bottom-quality firm keeps the bottom product (as a fighting brand) at the price stage whereas ultimately drops it at the quality stage. As argued above, keeping a fighting brand in an alliance is mostly a short-run (price) than a medium/long run strategy (quality) and it is, therefore, dropped when the merging group can re-position its product lines. Finally, it is found that, in all equilibrium (partial) mergers, the bottom-quality firm is always present. This appears in line with numerous theoretical and experimental studies on coalition formation in triads of heterogeneous individuals, i.e. possessing different skills or fighting ability (e.g. Caplow 1956, 1959, 1968, Vinacke and Arkoff 1957, Gamson 1961). A central conclusion of these studies is that "weakness is strength" (see, for instance, Mesterton-Gibbons *et al.* 2011, p.189), with this meaning that less-powered individuals have usually more chances to be part of a coalition.

The results of this coalition formation game confirms that the most likely mergers occur between intermediate and bottom-quality producers, with the premium quality brands preferably running alone. This is the case of some top car producers (as, for instance, Daimler-Benz) whose only participation is in a few specific projects. What the model results also indicate, is that mergers between intermediate and bottom quality firms, as those occurred between Volkswagen and Skoda, or between Fiat and Chrysler in the automotive industry, should be the norm. In these cases the intermediate quality product is withdrawn from the market, which can be interpreted saying that of all products sold by the merger have a tendency to converge towards the same level of quality of their premium brand products. The model also stresses how also the mergers between top and bottom quality firms are likely, as for instance those recently occurred between generics pharmaceutical manufacturers and premium brand pharmaceutical companies. The model results just suggest that in this case the bottom quality products can be profitably retired from the market to soften the competition among remaining goods.

## 7 Concluding Remarks

The rationale underlying many of the result presented in this paper can be found in the nature of competition among vertically differentiated firms. Indeed, in any cartel or merger, the optimal set of products to market is defined by balancing *the cannibalization effect* within the coalition with the *stealing effect* occurring between a coalition and the firms outside. It was shown that the bottom quality variant in a group of colluding firms is kept on sale in the market only when such a cartel needs it as a sort of *fighting brand* to protect itself from all lower quality variants sold by the fringe of competitors. In any other case a cartel prefers to withdraw from the market all its low quality variants. In this way firms can soften price competition in the market and magnify the quality differentiation between the variants remained on sale. This view seems in line with the empirical findings, where mergers emphasize "product differentiation" among merging firms as well as with respect to their outside rivals. Partial mergers can, therefore be viewed as a means to enhance the *dynamic* competition for the market and to reduce the *static* competition in the market.

## 8 Omitted Proofs

**Proof of Proposition 1.** (Gabszewicz et al. 2016a). Take a generic *intermediate* cartel of  $h \leq n - 2$  firms initially selling variants

$$q_k, q_{k+1}, q_{k+2}, \dots, q_{k+h}$$

and competing, both with a left-hand fringe of independent firms selling lower quality variants  $q_1, q_2, \dots, q_{k-1}$  and with a right-hand fringe selling, alternatively, higher quality variants  $q_{k+h+1}, q_{k+h+2}, \dots, q_n$ . Using expressions (14)-(16) the optimal-replies of the firms in the cartel are

$$\begin{aligned} p_k^{pc}(p_{k-1}, p_{k+1}) &= \frac{\frac{1}{2}p_{k-1}(q_{k+1} - q_k) + p_{k+1}(q_k - q_{k-1})}{(q_{k+1} - q_{k-1})} \\ p_{k+1}^{pc}(p_k, p_{k+2}) &= \frac{p_k(q_{k+2} - q_{k+1}) + p_{k+2}(q_{k+1} - q_k)}{(q_{k+2} - q_k)} \\ p_{k+2}^{pc}(p_{k+1}, p_{k+3}) &= \frac{p_k(q_{k+3} - q_{k+2}) + p_{k+3}(q_{k+2} - q_{k+1})}{(q_{k+3} - q_{k+1})} \\ &\dots\dots\dots \\ p_{k+h}^{pc}(p_{k+h-1}, p_{k+h+1}) &= \frac{p_{k+h-1}(q_{k+h+1} - q_{k+h}) + \frac{1}{2}p_{k+h+1}(q_{k+h} - q_{k+h-1})}{q_{k+h+1} - q_{k+h-1}}. \end{aligned}$$

where only the two extreme firms  $k$  and  $k + h$  in the cartel are directly competing with the firms outside. Without loss of generality, take a generic firm inside the cartel producing an intermediate variant (i.e neither the bottom nor the top quality within the cartel), say firm  $k + 1$ . Using both the optimal reply of firm  $k + 1$  and those of the firms connected to it (i.e. firms  $k$  and  $k + 2$ ) and re-arranging, we obtain the optimal replies of these three firms as functions of  $p_{k-1}$  and  $p_{k+3}$  only.

$$\begin{aligned} \tilde{p}_k &= p_k^{pc}(p_{k-1}, p_{k+3}) = \frac{1}{2} \frac{p_{k-1}(q_{k+3} - q_k) + 2p_{k+3}(q_k - q_{k-1})}{q_{k+3} - q_{k-1}}, \\ \tilde{p}_{k+1} &= p_{k+1}^{pc}(p_{k-1}, p_{k+3}) = \frac{1}{2} \frac{p_{k-1}(q_{k+3} - q_{k+1}) + 2p_{k+3}(q_{k+1} - q_{k-1})}{q_{k+3} - q_{k-1}}, \\ \tilde{p}_{k+2} &= p_{k+2}^{pc}(p_{k-1}, p_{k+3}) = \frac{1}{2} \frac{p_{k-1}(q_{k+3} - q_{k+2}) + 2p_{k+3}(q_{k+2} - q_{k-1})}{q_{k+3} - q_{k-1}}. \end{aligned}$$

Using the above, we can easily compute the optimal market share of firm  $(k + 1)$  as

$$D_{k+1}(\tilde{p}_k, \tilde{p}_{k+1}, \tilde{p}_{k+2}) = \frac{\tilde{p}_{k+2} - \tilde{p}_{k+1}}{q_{k+2} - q_{k+1}} - \frac{\tilde{p}_{k+1} - \tilde{p}_k}{q_{k+1} - q_k} = 0$$

which proves that under partial collusion every intermediate firm of an *intermediate* cartel obtains zero market share. Repeating now the same procedure for the firm producing the lowest quality in the cartel (here firm  $k$ ), we obtain instead that

$$D_k(\tilde{p}_k, \tilde{p}_{k+1}, \tilde{p}_{k-1}) = \frac{\tilde{p}_{k+1} - \tilde{p}_k}{q_{k+1} - q_k} - \frac{\tilde{p}_k - \tilde{p}_{k-1}}{q_k - q_{k-1}} = \frac{1}{2} \frac{\tilde{p}_{k-1}}{(q_k - q_{k-1})} > 0$$

for  $\tilde{p}_{k-1} > 0$ . Finally, computing the optimal replies of the highest quality firm in the cartel, i.e. firm  $(k+h)$ , and of the firms directly connected to it, we obtain

$$\begin{aligned} \tilde{p}_{k+h-1}(p_{k+h-2}, p_{k+h}) &= \frac{p_{k+h-2}(q_{k+h-1} - q_{k+h-2}) + p_{k+h}(q_{k+h-1} - q_{k+h-2})}{q_{k+h} - q_{k+h-2}} \\ \tilde{p}_{k+h}(p_{k+h-1}, p_{k+h+1}) &= \frac{p_{k+h-1}(q_{k+h+1} - q_{k+h}) + \frac{1}{2}p_{k+h+1}(q_{k+h} - q_{k+h-1})}{q_{k+h+1} - q_{k+h-1}} \\ \tilde{p}_{k+h+1}(p_{k+h}, p_{k+h+2}) &= \frac{\frac{1}{2}p_{k+h}(q_{k+h+2} - q_{k+h+1}) + p_{k+h+2}(q_{k+h+1} - q_{k+h})}{q_{k+h+2} - q_{k+h}}. \end{aligned}$$

Using the above,

$$\begin{aligned} D_{k+h}(\tilde{p}_{k+h-1}, \tilde{p}_{k+h}, \tilde{p}_{k+h+1}) &= \frac{\tilde{p}_{k+h+1} - \tilde{p}_{k+h}}{q_{k+h+1} - q_{k+h}} - \frac{\tilde{p}_{k+h} - \tilde{p}_{k+h-1}}{q_{k+h} - q_{k+h-1}} = \\ &= \frac{1}{2} \frac{\tilde{p}_{k+h+1}}{(q_{k+h} - q_{k+h-1})} > 0. \end{aligned}$$

showing that only the variants produced by the two firms at the extremes of this (generic) intermediate cartel are sold at prices implying *positive* market shares. Exactly the same procedure proves that, in a *top cartel*, only the highest and the lowest quality variants initially sold by the cartel remain on sale.

Finally, let us consider a *bottom cartel*, i.e. a cartel formed by firms  $1, 2, \dots, h$  initially selling  $h$  variants  $q_1, q_2, \dots, q_h$  and competing with  $(n-h)$  independent firms selling the higher quality variants  $q_{h+1}, q_{h+2}, \dots, q_n$ . Again, we can apply the same argument used above to show that every firm in the *interior* of the cartel (i.e. neither selling the lowest quality nor the highest quality variant in the cartel) obtains zero market share. Also, for the top quality firm in the cartel (here firm  $h$ ), we obtain that  $D_h(\tilde{p}_h, \tilde{p}_{h-1}, \tilde{p}_{h+1}) > 0$ . Finally, when considering a firm selling the lowest quality variant in any *bottom* cartel, its market share simply writes as:

$$D_1(p_2, p_1) = \frac{p_2 - p_1}{q_2 - q_1} - \frac{p_1}{q_1},$$

that, using firm's 1 optimal collusive reply  $p_1^{pc}(p_2) = \frac{q_1}{q_2}p_2$ , becomes



$$D_1(p_2, \tilde{p}_1) = \frac{p_2 - \frac{q_1}{q_2} p_2}{q_2 - q_1} - \frac{\frac{q_1}{q_2} p_2}{q_1} = 0,$$

showing that, differently from other cartels, a *bottom cartel* optimally produces only its top-quality variant  $q_h$ . **Q.E.D.**

**Proof of Proposition 4.** Under equispaced variants, from (6), for all  $k = 1, 2, \dots, n$  best-replies are

$$p_k = \frac{1}{4}(p_{k+1} + p_{k-1})$$

which can be written as a second-order difference equation as

$$p_{k+1} - 4p_k + p_{k-1} = 0,$$

with complementary function

$$Ab^{k+1} - 4Ab^k + Ab^{k-1} = 0.$$

and whose associated characteristic function possesses two distinct real roots given by

$$b_1 = 2 + \sqrt{3}, b_2 = 2 - \sqrt{3},$$

implying

$$p_k = A_1 b_1^k + A_2 b_2^k. \tag{20}$$

Moreover, using the fact that for the bottom quality firm,

$$p_1 = \frac{1}{4}p_2 = \frac{1}{4}(p_2 + p_0)$$

we can set

$$p_0 = A_1 b_1^0 + A_2 b_2^0 = A_1 + A_2 = 0$$

implying

$$A_2 = -A_1. \tag{21}$$

Finally, using the fact that for the top quality firm

$$p_n = \frac{1}{2}(p_{n-1} + \beta\tau)$$

we just write

$$2p_n - p_{n-1} = \beta\tau$$

that implies

$$p_{n-1} = A_1 b_1^{n-1} + A_2 b_2^{n-1} = A_1 (b_1^{n-1} - b_2^{n-1}) = 2A_1 (b_1^n - b_2^n) - \beta\tau$$

from which

$$A_1 (b_1^{n-1} - b_2^{n-1}) - 2A_1 (b_1^n - b_2^n) + \beta\tau = 0$$

and, then,

$$A_1 = \frac{\beta\tau}{(2b_1^n - 2b_2^n - b_1^{n-1} + b_2^{n-1})}$$

As a final step, we insert coefficients  $A_1$  and  $A_2$  in (20), obtaining

$$p_k^* = A_1 (b_1)^k + A_2 (b_2)^k = A_1 (b_1)^k - A_1 (b_2)^k = \frac{\beta\tau (b_1^k - b_2^k)}{\sqrt{3}b_1^n + \sqrt{3}b_2^n},$$

for every  $k = 1, 2, \dots, n$  and  $b_1 = (2 + \sqrt{3})$  and  $b_2 = (2 - \sqrt{3})$ , which concludes the proof.

**Q.E.D.**

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