Abstract

Currently researchers face a challenge when explaining how oligopolies operate on the market. The Cournot and Bertrand models provide an excellent starting point for the analysis of the competition between oligopolies. As a number of observed oligopolies is regulated by market forces and they co-exist with a myriad of small firms, a more realistic description of their co-existence requires other tools. In this paper, we introduce a simple new theory on the mixed competition between oligopolistic firms and the competitive fringe, assuming a comparative advantage of big firms and the free entry of small firms. Oligopolies are defined as conglomerates, each part of which benefits from joint operations through smaller costs. Our theory implies that (i) industries with a few oligopolies arise as a stable outcome of the mixed competition; (ii) the mixed competition differs from the monopolistic competition of single-product firms due to the underproduction of oligopolistic firms as well as differs from the pure oligopolistic competition since constraints on this underproduction are imposed by the competitive fringe; (iii) a positive shock in the market size strengthens the competitiveness of the economy through the growth in the number of oligopolies.

JEL Classification: D4, L10, F11

Keywords: mixed market structure; monopolistic competition; oligopolistic firms; general equilibrium; market size

Two and a Half Oligopolies in the Sea of the Competitive Fringe

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1 Introduction

Industries are typically divided into groups of big firms and a competitive fringe of much smaller firms [Porter, 2008, Uslay et al., 2010]. N. G. Mankiw (2007; Page 348) gives two following examples referring to the USA in 2002: (i) three companies controlled nearly two-thirds of the cable television market; (ii) three big publishers of college textbooks accounted for 65% of the industry. We can continue to find examples regarding different countries, industries and years, including the beef/sheep meat processing industry in Australia in 1987; three oligopolies — Kelloggs, Sanitarium and Nabisco — held a share of about 90% of the cereal market, and up to 30% of the wine industry was controlled by Adsteam and Philipp Morris [Lawrence, 1987].

This concentration of industries in the hands of several oligopolies gives rise to concerns regarding the efficiency of the industry itself. However, economic theory has investigated the strategic games of oligopolies and the monopolistic competition of the myriad of small firms separately. The principles of the co-existence of large and small firms is rarely discussed; for example, Gabszewicz and Vial [1972] introduced an analysis from the perspective of cooperative game theory. This gap in the literature has been filled in recent years [Neary, 2003, Etro, 2008, Neary, 2010, Shimomura and Thisse, 2012, Parenti, 2017, Kokovin et al., 2017].

We construct a new simple theory on the co-existence of oligopolies and the competitive fringe, assuming the comparative advantage of big firms and the free entry of small firms. Oligopolies are defined as conglomerates, each part of which benefits from joint operations thanks to smaller costs. This assumption follows the saying by Demsetz [1973]: "Under the pressure of competitive rivalry, and in the apparent absence of effective barriers to entry, it

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would seem that the concentration of an industry's output in a few firms could only derive from their superiority in producing and marketing products or in the superiority of industry in which there are only a few firms". We summarize our main findings in three points.

1. Industries with a few oligopolies arise as a stable outcome of the mixed competition between oligopolistic firms and the competitive fringe.

2. The mixed competition differs from the monopolistic competition of single-product firms due to the underproduction of oligopolistic firms. It differs from pure oligopolistic competition due to the constraints on underproduction imposed by the competitive fringe. Thus, the mixed competition looks like "a stage, where every man must play a part"[1]: oligopolies establish their market power, whereas small firms restrain it. In contrast to Shakespeare's characters, both sides benefit from participating.

3. A positive shock in the market size strengthens the competitiveness of the economy through the growth in the number of oligopolies.

We model the following framework of the mixed competition between small and big firms. All firms produce varieties of a differentiated good. Small firms are single-product. They have a local market power as modeled by Dixit and Stiglitz [1977]. Big firms have global market power. They produce a range of varieties, behave like monopolists on the market of the varieties produced by them, and, additionally, affect the price index associated with the differentiated good. According to their decision making, small firms are price-index takers, whereas big firms are price index-makers.

We find that each part of a conglomerate is better off when keeping its comparative advantage: it deviates from the output-pricing policy of the conglomerate and behaves in line with the strategy of single-product firms. When we introduce a fixed penalty for the deviation, the profit per variety of big firms equals to this penalty in equilibrium. The idea of the deviation costs is related to the prisoner's dilemma and is extensively used in the theory of conflicts.

The principles of the coexistence of small and big firms are derived under the constant elasticity of substitution (CES) preferences of consumers. Then we verify the general nature of the derived principles with unspecified separable preferences. With this type of preferences, we assess the market size effect.

Related literature. Dixit and Stiglitz [1977] introduce an approximation for the number of discrete varieties that altogether constitute a differentiated good. This approximation works well if each firm controls a negligible share of the market. Therefore, by specifying the range of shares between negligible and significant shares, one could discuss the behavior of big and small firms. Yang and Heijdra [1993], d'Aspremont et al. [1996] refine the Dixit–Stigitz approximation, thus dealing with less negligible but still small firms. Big firms can also be associated

[1] W. Shakespeare, The Merchant of Venice

with multi-productivity, as analyzed in details by Bernard et al. [2012], Dhingra [2013], Mayer et al. [2014], Eckel and Neary [2010], Ushchev [2017], as well as Feenestra and Ma [2007], and considered to be leaders in the Stackelberg game (see Etro [2008]) or giants on local markets that are small on the global market [Neary, 2003]. According to [Neary, 2010], firms can choose between being large or small in order to maximize their profits.

Recently, groups of authors have expanded the Dixit–Stiglitz approach to model a mixed market structure with big and negligibly small firms. From a measure-theoretic point of view, small firms were associated with points of measure zero on the segment that represents the differentiated good, whereas the number of varieties produced by big firms had a positive measure. Shimomura and Thisse [2012] tackled the competition of oligopolies, which produce a discrete set of varieties, and the myriad of negligibly small firms that form the competitive fringe. These authors investigated how the entrance of additional oligopolies affects the economy and, in particular, the mixed market structure. In the model, the competitive fringe behaves as an additional big firm. The number of oligopolies is an exogenous parameter; With its growth, the competitive fringe shrinks and finally disappears. Dixit [1979] predicted a blockaded entry of a weaker competitor in the duopoly setting, however the existence of pure oligopolistic competition in the modeling strategy by Shimomura and Thisse [2012] with an endogenous number of oligopolies is still unclear. In contrast to [Shimomura and Thisse, 2012], we avoid an atomic representation of big firms' output in favor of a continuous range of varieties produced by each oligopoly. Such a choice simplifies the analytics while keeping qualitative outcomes. Assuming the free entry of oligopolies, we estimate their number for different model parameters.

When there is a more significant comparative advantage, less oligopolies operate on the market, but they control a larger share of the varieties. We find an unexpected absence of equilibrium within oligopolies under their large comparative advantage. In this case, the equilibrium number of oligopolies is less than one and the mixed competition becomes unstable.

The approach designed by Parenti [2017] is closer to our own. In [Parenti, 2017], as in our analysis, large multi-product firms produce a positive share of varieties. By using the quadratic utility of consumers [Ottaviano et al., 2002], he discovers that a decrease in trade barriers increases the number of oligopolistic firms. In this paper, we establish a similar effect: in response to a sudden enlargement of the economy, industries become less oligopolistic. This result is obtained within the framework of a small economy and separable unspecified preferences. Following [Parenti, 2017] but not [Gabszewicz and Vial, 1972, d'Aspremont et al., 1990], we treat all firms as income-takers when they neglect the impact that their output decisions have on the total income through the distribution of profits.

Kokovin et al. [2017] introduced their mixed market structure of infinitesimally small single-product firms and big firms that produce a scope of varieties. In their model, big firms benefit from “hiding” their ability to affect market aggregates by copying the pricing policy of small firms. The existence of a common scalar aggregate attribute capturing cross-price effects in the demand system leads to the dilution of big firms’ market power [Kokovin et al., 2017]. In the case of homogeneous production, the dilution results in an identical pricing system and the profit equalization. If not, a less successful firm will mimic the prices and output of more efficient rivals.

Unlike Kokovin et al. [2017], we explain why only a few oligopolies operate on the market, relating this phenomenon to their comparative advantage. We posit that being endowed by a substantial comparative advantage, the existing oligopolies produce a wide range of varieties, thus, preventing the appearance of the other such wide-ranged competitors. As soon as oligopolies have a comparative advantage, their pricing policies differ from that of the competitive fringe. In contrast to Kokovin et al. [2017], we posit that oligopolies prefer to exploit their comparative advantage in spite of constraints on their market power through the demand system.

The underproduction and comparative advantage of big firms are related to the literature regarding heterogeneity of firms and total factor productivity [Melitz, 2003, Bernard et al., 2007, Redding, 2011, Hottman et al., 2016]. Melitz and Redding [2012] claim that more productive firms set higher markups. Our model predicts the same conclusion, but the mechanism is different: the strategic behaviour, meaning the possibility to manipulate the market, leads to the higher markups of oligopolies. With higher markups, big firms have enough room for an active pricing policy. On the contrary, tiny markups give small firms limited room for strategic adjustments.

The rest of the paper is organized in the following way. The economy is modelled in Section 2. We construct equilibrium and discuss its properties in Section 3. Section 4 concludes. All technical parts are placed into two Appendices.

2 Model

2.1 Economy

We consider a single-sector economy that produces a differentiated good. The production side is represented by single and multi-product firms. Multi-product firms operate as conglomerates of single-product firms. Their profits are shared between all individuals equally. Associating varieties of the differentiated good, which has the mass N, with the points of segment $[0, N]$, we prescribe points — that have the measure 0 — to single-product firms and segments — sets of a non-zero measure — to multi-product firms. The length of these segments indicate the scope of the varieties produced by multi-product firms. We also call the two type of firms small and large. When few big firms operate on the market they are associated with oligopolies. The set of small firms is frequently referred to as the competitive fringe.

Labor is a single production factor. Small firms hire homogeneous workers. The production of big firms is more complicated, thus requiring managers in addition to workers. The number of workers and managers in the economy is exogenous. For the sake of simplicity, the wages of both types of labor force are assumed to be equal and fixed to 1 as *numeraire*.

Individuals are homogeneous as consumers. They are endowed by separable unspecified utility, which is another exogenous characteristic of the economy.

2.2 Demand

An economy is populated by L consumers with income Y. A consumer chooses the quantities Q_x of the varieties $x \in [0, N]$ in order to maximize the utility

$$U = \int_0^N u(Q_x)\, dx \to \max \tag{1}$$

under budget constrains

$$\int_0^N p_x Q_x \leq Y, \tag{2}$$

where p_x is the price for the x-th variety of the differentiated good. The first order condition implies that

$$u'(Q_x) = \lambda p_x, \tag{3}$$

where λ is the Lagrange multiplier corresponding to optimization problem (1).

We will later show that the optimal demand and general equilibrium are described in terms of

$$\sigma(Q) = -\frac{u'(Q)}{u''(Q)Q}, \tag{4}$$

which is interpreted as the elasticity of substitution between varieties of the differentiated good.

The consumer's problem as formulated here is standard in monopolistic competition theory. We only note that consumers are indifferent of what a firm — large or small — produces the variety.

2.3 Supply

2.3.1 Small firm

A small firm producing the variety x maximizes its profit

$$\pi_{S,x} = (p_{S,x} - c_{S,x})q_{S,x} - F_{S,x} \to \max \tag{5}$$

where the prices $p_{S,x}$ and the output $q_{S,x}$ are the optimization variables. In the optimum, the output $q_{S,x} = LQ_{S,x}$ is equal to the aggregate demand for the variety x and the prices are

$$p_S = \frac{\sigma_S c_S}{\sigma_S - 1}, \tag{6}$$

where the index x is dropped and $\sigma_S = \sigma(Q_S)$ [Dixit and Stiglitz, 1977].

2.3.2 Big firm

A big firm produces a scope of varieties (of mass $N_B > 0$) and competes with the other firms. Its profit is

$$\Pi = \int_0^{N_B} \pi_{B,x}\, dx \to \max \tag{7}$$

where

$$\pi_{B,x} = (p_{B,x} - c_{B,x})q_{B,x} - F_{B,x} \tag{8}$$

and $q_{B,x} = LQ_{B,x}$ is the aggregate demand for the variety x. As Equations (5) and (8) read, the profit per variety of a big firm is structured in the same way as the profit of a small firm.

We recall that a small firm maximizing its profit does not affect integral market characteristics. We assume that a big firm does affect them. In particular, the Lagrange multiplier λ, Equation (3), depends on the range of prices chosen by a big firm[2]. In the case of the CES preferences, the Lagrnage multiplier is related to the price index of the differentiated good. Hence, big firms behave as price-index makers, whereas small firms behave as price-index takers. Under monopolistic competition, all firms are price makers, and the difference between strategic — big — and non-strategic — small — firms is observed through their influence on the price index.

For the sake of simplicity, we imply a symmetric setting among both types of firms. In particular, the costs $c_{S,x}$ and $F_{S,x}$ are independent from x. The first order condition of a big firm's problem relates the price $p_{B,x}$ being charged by this big firm to its variable costs $c_{B,x}$ and output Q_B. Further simplifying the optimization problem, we look for the symmetrical pricing policy of each big firm: the prices $p_{B,x} = p_B$ do not depend on x. Then[3] the first order condition leads to

$$p_B = \frac{\sigma_B c_B}{\sigma_B - 1} \cdot \frac{1}{1 - p_B N_B Q_B}, \tag{9}$$

[2] Technically, the maximization in (7) is performed with respect to the range of prices, i.e. with respect to the function p_x. When computing the variation of the profit, we involve the Gateaux derivative. The Lagrange multiplier λ is "hidden" in the aggregate demand q_x. Its Gateaux derivative is found with a standard variation technique (see Appendix) whereas this derivative is assumed to be zero for small firms after Dixit and Stiglitz.

[3] Technically, constant functions are substituted into the first order condition written with the Gateaux derivatives; see the details in the Appendix.

where $\sigma_B = \sigma(Q_B)$; see Lemma 5. Equation (9) differs from (6) by the multiplier $1/(1-pN_BQ_B)$, where pN_BQ_B is the share of the income spent by the consumer on the total output of the big firm. This multiplier shifts the solution p_B of Equation (9) up. The corresponding growth of the optimal demand Q_B keeps $p_BN_BQ_B$ separated from 1. Hence, by affecting the price index, big firms decrease their output and charge higher prices. In Lemma 6 in the Appendix, we establish that the solution p_B of Equation (9) satisfies the second order conditions and, therefore, does maximize the profit Π.

2.3.3 Pricing policies of big and small firms

Economic forces stay behind the existence of big firms. We do not model this process, accepting it as it is. However, following Demsetz [1973] among others, we assume that big firms have a comparative advantage in costs: $c_B < c_S$, $F_B = F_S = F$. A model with different fixed costs leads to similar qualitative conclusions. We set the simplest dependence of the variable costs on the firm mass. Namely, if a variety is produced by a big firm instead of a small firms, then the marginal production costs are reduced from c_S to c_B[4].

According to Equations (6) and (9), the pricing policies of big and small firms are different. Indeed, in contrast to small firms, a big firm affects market aggregates and, as a result, decreases the output charging higher prices. More precisely, if a small firm faced the same costs as big firms do, it would set a lower price, Equation (6) vs. (9). Endowed by a larger market power, big firms produce less and charge higher prices than single-product firms do. In other words, the strategy of big firms is more monopolistic than that of small firms. We emphasis that an oligopoly is tempted to behave less monopolistic and let prices down in order to force the exit of the competitive fringe that operates at zero profit. However, such a strategy only renews the competitive fringe still keeping it on the market under free entry.

We note that under identical costs, big and small firms should decide upon their price and output identically in equilibrium: $p_S = p_B$; otherwise, a less profitable rival copies the strategy of the competitors. This causes the conglomerates of small firms to be unstable. Any part of a conglomerate can be separated without any impact on individual agents or the whole economy.

2.4 Balances

2.4.1 Acceptance to agglomerates and free entry

The process of firms' formation is beyond the scene. Our model is static. Considering a big firm as a conglomerate of infinitesimally small single-product parts, we emphasis that each

[4] If the mass of a firm is 0, its costs are c_B; if the mass is positive then they are c_S; the value of c_B does not depend on the positive firm mass.

part has incentives to deceive by deviating from the firm's strategic price-output policy. Indeed, benefiting from the comparative advantage in costs, a single-product part maximizes its profits when producing more and charging less for its output in line with (5). However, the deviation has to be subject for penalties up to the exclusion from the conglomerate. Simplifying this process and following the literature on club formation, see, for example, [Alesina and Spolaore, 1997, Bolton and Roland, 1997], we consider these penalties as the deviation cost, to which the value $\varphi \geq 0$ is assigned. Then, the profit of a big firm per variety $\pi_{B,x}$, where x belongs to the scope of the big firm, is equal to φ in equilibrium, as φ is the largest admissible profit that rules out the deviations. We also assume that big firms are identical, and n denotes their number.

Small firms are free to enter the market. Therefore, their profit is zero $\pi_{S,x} = 0$ in equilibrium; x labels the varieties produced by small firms.

2.4.2 Labor market clearance

We assume that the shares θ_W and θ_M of workers and managers in the economy are given. Since managers are employed only by big firms, it follows that the fixed costs F coincide with the number of managers in a big firm, and the total number of managers $\theta_M L$ is equal to

$$\theta_M L = n N_B F. \tag{10}$$

3 Equilibrium

3.1 Definition

The variables θ_M, θ_W, L, $u(\cdot)$, c_S, c_B, F, φ are exogenous in the model. We aim at finding the other characteristics of the economy. The set of the identical prices $\hat{p}_S = \hat{p}_{S,x}$ and $\hat{p}_B = \hat{p}_{B,x}$, outputs $\hat{Q} = \hat{Q}_x$ (all of them are independent of x), the mass of small firms $\hat{N}_S$, the mass $\hat{N}_B$ of each big firm, and the number $\hat{n}$ of big firms is called equilibrium if the following conditions hold.

First, the demand $\hat{Q}$ solves the consumer's optimization problem (1), (2) with $Y = 1 + \hat{n}\hat{N}_B\varphi/L$, $N = \hat{N}_S + \hat{n}\hat{N}_B$, $p_x = \hat{p}_S$ if the variety x is produced by a small firm and $p_x = \hat{p}_B$ otherwise.

Second, firms solve their optimization problems. Namely, given $N = \hat{N}_S + \hat{n}\hat{N}_B$, the output q_x is related to the prices p_x for this variety by solving the consumer's problem and is considered as a function of the prices in the small or big firm's optimization problem: Equation (5) or (7). The prices $\hat{p}_{S,x}$ and $\hat{p}_{B,x} = \hat{p}_B$ give the maximum of the profits (5) and (7), respectively. These prices enter into the profits directly and indirectly through q_x.

Third, balances (2), turned to the equality, and (10) hold with non-negative N_S and N_B.

Forth, all big firms have the same mass $\hat{N}_B$.

According to this definition, the equilibrium variables solve the system of equations (2), (3), (6), (9), and (10). We'll drop the diacritical mark ˆ from the notation in what follows.

3.2 Existence and uniqueness

The existence of the equilibrium requires the following assumption.

Assumption 1. *The function $\sigma(Q)$ is assumed to satisfy the following inequalities:*

$$\sigma(Q) > 1, \tag{11}$$

$$\sigma'(Q) \leq 0, \tag{12}$$

$$\frac{1-\theta_M \sigma(Q)}{\theta_M\big(\sigma(Q)-1\big)} > \frac{\varphi}{F}. \tag{13}$$

We highlight a special case of preferences satisfying the inequalities (11) and (12). It is represented by the CES utility functions

$$u(Q) = q^{\rho}, \quad \rho \in (0,1) \quad \sigma(Q) = 1-\rho = \text{const}.$$

These functions u indicate the frontier between the two classes of utilities that have increasing and decreasing elasticity of substitution as a function of Q. Inequality (12) means that we consider utilities from one of these classes as Krugman [1979] has done. Inequality (11) is technical.

Inequality (13) provides the labor market balance. If it is violated, then the number of managers is too big; under the balance on the labor market, big firms are forced to produce more products than consumers demand even if small firms are absent. Eventually, labor market equilibrium contradicts the balance between supply and demand.

Proposition 1. *Let Assumption 1 hold. Then equilibrium exists. Under CES preferences, equilibrium is unique.*

To simplify notation, we put $\sigma_S = \sigma(Q_S)$, $\sigma_B = \sigma(Q_B)$. In equilibrium, the outputs q_S and q_B of small and big firms are respectively equal to

$$q_S = \frac{F(\sigma_S - 1)}{c_S}, \tag{14}$$

$$q_B = \frac{F+\varphi}{c_B} \cdot \Big(\sigma_B - \frac{1}{2} - \frac{1}{2}\sqrt{1+4\sigma_B(\sigma_B-1)m}\Big) \cdot \frac{2}{1+\sqrt{1+4\sigma_B(\sigma_B-1)m}}, \tag{15}$$

where $m = N_B(F+\varphi)/L$ consists of the wages $N_B F/L$ of a big firm's managers, as normalized by the total mass of individuals and the contribution $N_B\varphi/L$ of a big firm's shares to the income of each individual; see Lemma 8. In Lemma 8 we establish that m solves the equation

$$m = \left(1 - \frac{(\sigma_S - 1)\sigma_B}{\sigma_S(\sigma_B - 1)} \cdot \frac{c_B}{c_S} \frac{u'(Q_S)}{u'(Q_B)}\right) \left(1 - \frac{\sigma_S - 1}{\sigma_S} \frac{c_B}{c_S} \frac{u'(Q_S)}{u'(Q_B)}\right). \tag{16}$$

We have discussed the mechanism of the underproduction of big firms in section 2.3.3. Based on Equation (15), which the output q_B satisfies, we rigorously establish the underproduction rigorously in Lemma 11.

The introduction of big firms complicates the standard rigorous analysis of the equilibrium equations. In particular, instead of the single Equation (14), which gives the output of small firms, the (closed) system of two Equations (15), (16) is required to expose the equilibrium characteristic of big firms. We establish the existence of equilibrium by exploring this system.

3.3 Mass of small firms

The budget constraint as well as the balance of money re-written with equilibrium variables turn into

$$\sigma_S F N_S + \frac{2\sigma_B(F + \varphi)N_B n}{1 + \sqrt{1 + 4\sigma_B(\sigma_B - 1)\frac{(F+\varphi)N_B}{L}}} = L + nN_B\varphi. \tag{17}$$

The mass N_S of small firms is computed with Equation (17), and, in general, can be negative. In this case, big firms push the competitive fringe out of the market. This effect is in line with theoretical predictions made by Dixit [1979]. Our approach can also describe the market without small firms, but the size asymmetry between oligopolies should be allowed. To avoid asymmetry of oligopolies within this paper, we introduce condition (13) that keeps the competitive fringe in equilibrium. Indeed, since the square root is positive, the inequality

$$1 + \theta_M \frac{\varphi}{F} - \sigma(Q_B)\theta_M \left(1 + \frac{\varphi}{F}\right) \geq 0 \tag{18}$$

together with (10) leads to $N_S \geq 0$; see (17). Inequality (18) follows from (13).

According to (13), three small quantities — the share of managers θ_M, the profit φ/F of big firms normalized by the fixed costs, and the elasticity of substitution $\sigma(\cdot)$ — stay in favor of the competitive fringe. A limited number of managers restricts the expansion of big firms, whereas low profits decrease their attractiveness. The inverse elasticity of substitution, $1/\sigma$, represents the inclination of consumers to the diversity of the differentiated good. The market creates a niche for the competitive fringe when consumers do prefer the diversity.

3.4 Case of a half of big firm

When constructing a model with a continuous set of firms, we can ignore the fact that the derived number of oligopolies is fractional until it exceeds 1. However, if the comparative advantage of a big firm is significant, then its scope is so huge that mathematical routine results in the value n, which is less than 1. It means that even a single oligopoly fails to find enough managers to run its production. Therefore, the expansion of a single big firm involves training additional managers.

3.5 Number of big firms

According to (10) and the definition of m, the number of big firms is given by $n = \theta_M(1 + \varphi/F)m^{-1}$. Since the number of firms n is at least 1, the quantity m should be small. The latter holds if both brackets in (16) are close to zero; in particular, if the ratio c_B/c_S is close to 1 and φ is negligible with respect to F. We associate the comparative advantage of big firms with the parameter ε, defined as $\varepsilon = 1 - c_B/c_S$. If ε is small, the approximate solution of Equation (16) can be found through the expansion of its right-hand side into series. This leads to an approximate formula for the number of firms.

We need another technical assumption to estimate the number of firms. The function $r_f(\varkappa) = -f''(\varkappa)\varkappa/f'(\varkappa)$ is assigned to the arbitrary function $f(\varkappa)$.

Assumption 2. *Let*

$$r_{u'}(Q_B) < 2. \tag{19}$$

We note that CES preferences satisfy Assumption (2).

Proposition 2. *Under CES preferences the number of big firms is*

$$n \approx \frac{\theta_M(F+\varphi)}{F}\left(\left(\frac{\sigma^3}{2}\right)^{1/2}\left(1 - \frac{c_B}{c_S} + \frac{\varphi}{(\sigma-1)F}\right)^{-1/2} - \frac{\sigma}{2} + 1 + O\left(\left(1 - \frac{c_B}{c_S} + \frac{\varphi}{F}\right)^{1/2}\right)\right) \tag{20}$$

Let an unspecified utility satisfy Assumptions 1 and 2. Then the number n of big firms as a function of $\varepsilon_\varphi = 1 - c_B/c_S + \varphi/F$ behaves in the following way:

$$n \approx \frac{\theta_M(F+\varphi)}{F}\left(\frac{\sigma^3(Q_B)(1 - r_u(Q_B))}{2(2 - r_{u'}(Q_B))}\right)^{1/2}\left(1 - \frac{c_B}{c_S} + \frac{\varphi}{(\sigma(Q_B)-1)F}\right)^{-1/2} + O(1). \tag{21}$$

where O-big term is taken with respect to ε_φ at the point $\varepsilon_\varphi = 0$.

The number of oligopolies positively correlates with the share of managers θ_M, the profit of big firms per variety normalized by the fixed costs φ/F, and the elasticity of substitution σ.

Figure 1 illustrates Equation (20). This Figure gives evidence that the model realizes the economy with a few number of oligopolistic firms for adequate values of the elasticity of substitution $\sigma \in [2, 5]$.

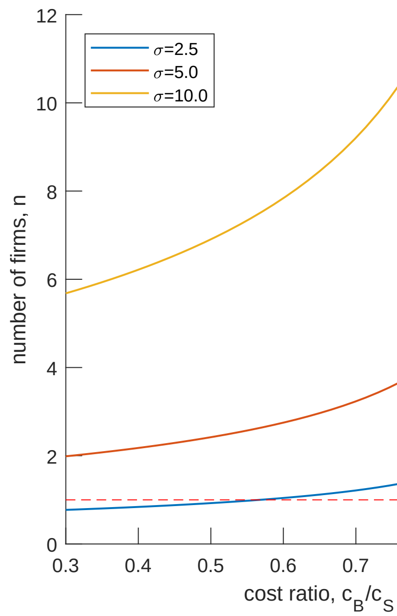

Figure 1: The number of firms found with Equation (20), $\varphi = 0$, $\theta_M = 0.25$, and three values of σ; the horizontal dashed line indicates 1.

The approximation (21) obtained for the economy with an unspecified utility is less accurate than (20), since only the main term with respect to $1/\varepsilon_\varphi$ is found, but the second term, a constant, is skipped. One would expect that the expansion into the Taylor series would result in the leading term being proportional to ε^{-1} in Equation (21). Nevertheless, the number of big firms increases as the square root of $\varepsilon^{-1/2}$ does, as the latter tends towards 0. This growth is relatively small; the market with a few oligopolistic firms can be observed with a wide range of comparative advantages. Merely the dependence on ε_φ is explored by Equation (21) rather than the presice value observed with a fixed ε_φ, since $O(1)$ has an order of the constant. We stress that the $(1 - c_B/c_S)^{-1/2}$-growth of n is a general characteristic of the model economy.

3.6 Comparative statics

In order to assess the market size effect we work with the unspecified utilities (1). We introduce an additional technical assumption on the utility.

Assumption 3. *Let $r_{u'}(Q)$ be a decreasing function of Q.*

When modeling monopolistic competition beyond CES, researchers involve examples of preferences, including the CARA utilities, that satisfy Assumption 3 [Behrens and Murata, 2012].

Proposition 3. *Let Assumptions 1–3 hold. Then approximation (21) to the number of large firms increases in L.*

A shock in the market size affects big firms in a natural way. With the increase of individuals, the demand for product diversity enlarges. This shrinks the market power of each firm. It implies that a big firm is more restricted on larger markets when exploiting its comparative advantage. In other words, the comparative advantage in costs loses its significance on larger markets. Therefore, by responding to a positive shock in the market size, big firms decrease their scopes. The share of their varieties becomes smaller. As the number of managers is assumed to be independent of L, Equation (10) implies that the number of conglomerates is greater on larger markets.

4 Concluding Remarks

We have constructed a simple theory of the mixed competition between oligopolies and the myriad of small firms. Still, several issues are worth developing further. In many examples, a big firm acts more like an indivisible unit than a conglomerate of weakly dependent parts. Such an oligopoly is free to optimize its scope. In other words, oligopolies gain an additional dimension of market power that leads to a drop in scope. This creates a force that increases the number of oligopolies.

We note that an indivisible firm can extend its scope when the expansion decreases its profit per variety because the maximization of the profit Π and its average Π/N_B with respect to the scope N_B clearly differ. Moreover, an oligopoly can further enlarge the scope even if it decreases the total profit. Indeed, assume that the production of a new variety is still profitable for a small firm. This attracts new small firms to enter the market. The appearance of a new small firm harms a big firm more than its own expansion, since the expansion allows the firm to compensate a loss in demand by "picking up" a positive profit that comes from launching a new variety. In this case, oligopolies exhibit a kind of cannibalism. The cannibalism creates an economic force that stays behind a lesser number of oligopolies.

Our analysis uncovers the incentives of oligopolies to deviate from a symmetrical output–pricing policy. We relate these incentives to the possibility of the secession of a firm's part. Alternatively, we could think about the asymmetrical policy of an indivisible oligopoly. This requires more sophisticated mathematics to tackle the corresponding optimization problem.

A Construction of Equilibrium

We prove the existence of general equilibrium and its uniqueness in the case of CES preferences, as stated in Proposition 1. The proof is performed in several lemmas presented one-by-one with brief comments regarding their content. The lemmas are integrated into the rigorous proof of

Proposition 1 at the end of this Appendix.

The first lemma solves the consumer's optimization problem.

Lemma 1. *Let a consumer facing varieties $x \in [0, N]$ traded at prices p_x maximizes her profit* (1) *under the budget constrain*

$$\int_0^N p_x Q_x \leq Y. \tag{22}$$

Then the optimal demand sought among interior solutions satisfies to the budget constrain (22) *written as the equality, Equation* (3)*, and*

$$\lambda = \frac{1}{Y} \int_0^N u'(Q_x) Q_x \, dx. \tag{23}$$

Proof. The first order condition of the maximization problem leads to (3). Substituting Equation (3) to the budget (22), which is understood as the equality, we obtain Equation (23). □

We turn to a firm's optimization. Its solution involves the variation of the aggregate and individual demands, $q_{B,x}$ and $Q_{B,x}$, with respect to the prices. To simplify notation, we drop index B in the proof.

Small firms control only their own prices. Therefore, the partial derivative represents the variation with respect to these prices. Big firms choose various prices; they are given by a function. In this case, the Gateaux derivative characterizes the variation. We limit ourselves by the symmetrical pricing policies: big firms charge identical prices for their products. This allows us to move from the Gateaux to the partial derivative. One can do it immediately, differentiating (3) with respect to p_x, where both of the multipliers of the right-hand side depend on the prices. Instead, we prefer to elaborate a general case, Lemmas 2–4, in hope of enlarging the toolkit of the monopolistic competition theory.

Lemma 2. *The first order condition of optimization problem* (7) *is given by*

$$(p_{B,x} - c_{B,x}) \frac{\delta q_{B,x}[p_x]}{\delta p_x} + q(p_{B,x}) = 0 \quad \forall x \in [0, N_B], \tag{24}$$

where quantities inside the square brackets indicate the (functional) variables of the outer functions.

Proof. Recall, the Gateaux derivative of the function Φ, $\Phi : X \to Y$ is defined in two steps. First,

$$\delta\Phi = \lim_{t \to 0} \frac{d}{dt} \frac{\Phi(x + th) - \Phi(x)}{t}.$$

If $\delta\Phi = G(x)h$, then the mapping $G(x)$ is called the Gateaux derivative denoted by $\frac{\delta Phi}{\delta x} = G$.

The variation of the profit Π is

$$\delta\Pi = \lim_{t\to 0}\frac{1}{t}\int_0^{N_B}\big(p_x\cdot(q_x[p_x+th_x]-q_x[p_x])+th_x\cdot q[p_x+th_x]-c_x(q_x[p_x+th_x]-q_x[p_x])\big)\,dx =$$
$$\int_0^{N_B}\left(p_x\frac{\delta q_x[p_x]}{\delta p_x}h_x+h_xq[p_x]-c_x\frac{\delta q_x[p_x]}{\delta p_x}h_x\right)\,dx = \int_0^{N_B}\left((p_x-c_x)\frac{\delta q_x[p_x]}{\delta p}+q[p_x]\right)h_x\,dx.$$

Since the variation of the profit is zero for any feasible[5] function h_x, we end up with (24). □

We are going to vary the optimal demand $Q_{B,x}$ with respect to the prices $p_{B,x}$. As an auxiliary computation, Lemma 3 represents the variation of the Lagrange multiplier λ.

Lemma 3. *The Gateaux derivative of λ, as determined by Equation* (23) *with respect to the prices $p(x)$ for varieties $x\in[0,N_B]$, is the linear operator*

$$\frac{\delta\lambda}{\delta p}h = \int_0^{N_B}K(p_x)h_x\,dx,$$

where

$$K(p_x) = \frac{1}{Y}\big(u''(Q_x(p_x))Q_x(p_x)+u'(Q_x(p_x))\big)\frac{\delta Q_x}{\delta p}h_x.$$

Proof.

$$\delta_p\lambda(p_x,h_x) = \lim_{t\to 0}\frac{1}{t}\frac{\lambda(p+th)-\lambda(p)}{t} =$$
$$\lim_{t\to 0}\frac{1}{t}\frac{1}{Y}\int_0^{N_B}\big(u'(Q_x(p_x+th_x))Q_x(p_x+th_x)-u'(Q_x(p_x))Q_x(p_x)\big)\,dx,$$

where the function h_x is zero outside the interval $[0,N_B]$. Simplifying, we get:

$$\delta_p\lambda(p_x,h_x) = \lim_{t\to 0}\frac{1}{t}\frac{1}{Y}\int_0^{N_B}\big(u'(Q_x(p_x+th_x))-u'(Q_x(p_x))\big)Q_x(p_x+th_x)+$$
$$u'(Q_x(p_x))\big(Q_x(p_x+th_x)-Q_x(p_x)\big)\,dx.$$

$$\delta_p\lambda(p_x,h_x) = \frac{1}{Y}\int_0^{N_B}\Big(u''(Q_x(p_x))\lim_{t\to 0}\frac{1}{t}\big(Q_x(p_x+th_x)-Q_x(p_x)\big)Q_x(p_x)+$$
$$u'(Q_x(p_x))\lim_{t\to 0}\frac{1}{t}(Q_x(p_x+th_x)-Q_x(p_x))\Big)\,dx.$$

$$\delta_p\lambda(p_x,h_x) = \frac{1}{Y}\int_0^{N_B}\big(u''(Q_x(p_x))Q_x(p_x)+u'(Q_x(p_x))\big)\frac{\delta Q_x}{\delta p}h_x\,dx.$$

Let

$$K(p_x) = \frac{1}{Y}\big(u''(Q_x(p_x))Q_x(p_x)+u'(Q_x(p_x))\big)\frac{\delta Q_x}{\delta p}h_x.$$

[5] We avoid the description of the functional spaces required for a rigorous formulation of the large firm's optimization problem.

Then the Gateaux derivative is the linear operator that maps the function h to $\mathbf{R}$, as stated in the Equation:

$$\frac{\delta\lambda}{\delta p}h = \int_0^{N_B} K(p_x)h_x\,dx.$$

□

Based on Lemma 3, we derive an integral equation with respect to $\delta Q/\delta p$ in the following Lemma.

Lemma 4. *Let*

$$I_2 = \int_0^{N_B} u''(Q[p])Q[p]\frac{\delta Q}{\delta p}h\,dx.$$

Then

$$\left(1 - \frac{1}{Y}\int_0^{N_B} pQ[p]\,dx\right) I_2 = \int_0^{N_B} \lambda hQ[p]\,dx + \frac{1}{Y}\int_0^{N_B} pQ[p]\,dx \int_0^{N_B} u'(Q[p])\frac{\delta Q}{\delta p}h\,dx. \quad (25)$$

Proof. We are going to vary Equation (3) established in Lemma 1 with respect to the prices p_x charged by a single big firm and, therefore, defined on $[0, N_B]$. Initially, we substitute $p_x + th_x$ for p_x and drop the dependence on x:

$$u'(Q[p + th]) = \lambda[p + th](p + th).$$

Adding and subtracting $\lambda[p](p + th)$, we have:

$$u'(Q[p + th]) = (\lambda[p + th] - \lambda[p])(p + th) + \lambda[p]p + \lambda[p]th, \quad (26)$$

Subtracting (3) from (26) we get:

$$u''(Q[p])\delta Q[p]th = \lambda[p]th + (\delta\lambda[p])(p + th).$$

By using Lemma 3, we tend t to zero:

$$u''(Q)\frac{\delta Q}{\delta p}h = \lambda h + \frac{p}{Y}\int_0^{N_B} \left(u''(Q(p))Q(p) + u'(Q(p))\right)\frac{\delta Q}{\delta p}h\,dx,$$

where all functionals are defined in p. Multiplying by Q and integrating both parts of the last inequality over the interval $[0, N_B]$, we have

$$I_2 = \int_0^{N_B} \lambda hQ[p]\,dx + \frac{1}{Y}\int_0^{N_B} pQ[p]\,dx \left(I_2 + \int_0^{N_B} u'(Q[p])\frac{\delta Q}{\delta p}h\,dx\right).$$

□

From now on, we limit ourselves to the consideration of the symmetrical case: big firms choose identical prices on each variety of their scopes. According to the following lemma, this assumption drastically simplifies Equation (25) which determines the variation of the optimal demand with respect to prices. As a result, we find the optimal price that maximizes the profits of a big firm.

Lemma 5. *Let N_B be the total mass of varieties produced by a single big firm. We assume that the prices of its varieties are symmetrical: $p_{B,x} = p_B = \text{const}$. Then*

$$\frac{p_B - c_B}{p_B} = \frac{1}{\sigma_B} + \frac{p_B N_B Q}{Y} \frac{\sigma_B - 1}{\sigma_B}. \tag{27}$$

Proof. We drop the index B to simplify notation. Let $p_x = \text{const}$, $Q_x = \text{const}$, $h_x = 1$. Then the symbol δ of the variation can be changed to ∂, which stays for the partial derivative. From (25) it follows that

$$u''(Q)Q\frac{\partial Q}{\partial p}\left(1 - \frac{pQN_B}{Y}\right) = \lambda Q + \frac{pQN_B}{Y}u'(Q)\frac{\partial Q}{\partial p}.$$

With the definition (4) of $\sigma(Q)$ and the expression (3) for λ, the last equation implies that

$$\left(1 + \frac{pQN_B(\sigma - 1)}{Y}\right)\frac{\partial Q}{\partial p} = -\frac{\sigma Q}{p}. \tag{28}$$

From (24) and equation $q = N_B Q$, it follows that $\partial Q/\partial p = -Q/(p - c)$. Combining this observation with (28), we have (27). □

Lemma 6. *We consider a profit Π of a big firm as a function of prices p_B. These prices does not depend on the variety type. The variation of Π with respect to the prices is changed to the partial derivative. Let Assumption 1 hold. Then $\partial^2\Pi/\partial p_B^2 < 0$ at a point that satisfies the first order condition* (9).

Proof. Simplifying the notation of the proof, we drop the index B and let $p = p_B$, $Q = Q_B$, $N = N_B$. As derivatives substitute variations, the second derivative of the profit is given by

$$\Pi'' = N_B L\frac{\partial Q}{\partial p}\left(2 + (p - c)\frac{\partial^2 Q/\partial p^2}{\partial Q/\partial p}\right). \tag{29}$$

We find the second derivative of the demand by computing the derivatives of both sides of Equation (3) and by using the derivative of λ given by Lemma 3:

$$\frac{\partial Q}{\partial p} = -\frac{\sigma Q}{p}\frac{1}{1 + NpQ(\sigma - 1)/Y}. \tag{30}$$

The alternative way of getting this equation is to simplify (25) with $p = \text{const}$ and $h = 1$. Then taking the logarithm and computing the derivative of both sides of the obtained equation, we have:

$$\frac{\partial^2 Q/\partial p^2}{\partial Q/\partial p^2} = -\left(\frac{\sigma'}{\sigma} + \frac{1}{Q}\right)\frac{\partial Q}{\partial p} + \frac{1}{p} + \\ \frac{N}{Y + NpQ(\sigma - 1)}\left(Q(\sigma - 1) + p(\sigma - 1)\frac{\partial Q}{\partial p} + pQ\sigma'\frac{\partial Q}{\partial p}\right). \tag{31}$$

Understanding symbol $\sim$ as proportionality and substituting (31) into (29), we have:

$$-\Pi'' \sim 2-(p-c)\left(\frac{\sigma'}{\sigma}+\frac{1}{Q}\right)\frac{\partial Q}{\partial p}+\frac{(p-c)Np(\sigma-1+Q\sigma')}{Y+NpQ(\sigma-1)}\frac{\partial Q}{\partial p}+\frac{p-c}{p}+\frac{(p-c)NQ(\sigma-1)}{Y+NpQ(\sigma-1)}.$$

Taking into account (27), we sum up the last two terms on the right-hand side and get:

$$-\Pi'' \sim 2-(p-c)\left(\frac{\sigma'}{\sigma}+\frac{1}{Q}\right)\frac{\partial Q}{\partial p}+\frac{(p-c)Np(\sigma-1+Q\sigma')}{Y+NpQ(\sigma-1)}\frac{\partial Q}{\partial p}+\frac{Y+2NpQ(\sigma-1)}{\sigma}.$$

With a big firm's first order condition $\frac{\partial Q}{\partial p}=-\frac{Q}{p-c}$, which follows from (24), we simplifty Π'' to

$$-\Pi'' \sim 2+\left(\frac{\sigma'}{\sigma}+\frac{1}{Q}\right)Q-\frac{Np(\sigma-1+Q\sigma')Q}{Y+NpQ(\sigma-1)}+\frac{1+2NpQ(\sigma-1)/Y}{\sigma}.$$

We group the terms in the following way:

$$\Pi''=\left(2+\frac{\sigma'Q}{\sigma}\right)+\left(1-\frac{NpQ(\sigma-1)}{Y+NpQ(\sigma-1)}+\frac{1+2NpQ(\sigma-1)/Y}{\sigma}\right)+\left(\frac{Np(-\sigma')Q^2}{Y+NpQ(\sigma-1)}\right).$$

The condition (12) provides that the first and third brackets are positive. Establishing that the second bracket is positive, we put $t=NpQ(\sigma-1)/Y$ and prove the inequality

$$\frac{2t^2-(\sigma-3)t+1}{(1+t)\sigma}+1>0.$$

Since $\sigma>1$, it is enough to prove that $2t^2+3t+2>0$. The latter is evident. $\square$

Lemma 7. *The output of a small firm under the free entry ($\pi_S=0$) is given by Equation* (14).

The proof of the Lemma is well known.

Lemma 8. *Under the zero profit condition for a big firm, its prices and outputs are given by the following equations*

$$p_B=c_B\sigma_B\frac{2}{2\sigma_B-1-\sqrt{1+4\sigma_B(\sigma_B-1)m}}, \tag{32}$$

$$Q_B=\frac{(F+\varphi)(\sigma_B-1)}{Lc_B}\cdot\frac{\sigma_B-\frac{1}{2}-\frac{1}{2}\sqrt{1+4\sigma_B(\sigma_B-1)m}}{\sigma_B-1}\cdot\frac{2}{1+\sqrt{1+4\sigma_B(\sigma_B-1)m}}, \tag{33}$$

where m solves Equation (16).

Proof. Equalizing the profit (8) per variety of a big firm to φ we get

$$Q_B=\frac{F+\varphi}{p_BL}\left(1-\frac{c_B}{p_B}\right)^{-1}=\frac{F+\varphi}{(p_B-c_B)L} \tag{34}$$

Substituting (34) into (27), we get the equation

$$p_B\left(1-\frac{p_B(F+\varphi)N_B}{p_B-c_B}\right)=\frac{\sigma_Bc_B}{\sigma_B-1},$$

which is quadratic with respect to p_B. By dividing both sides by p_B and transforming, we obtain

$$1 - \frac{\sigma_B}{\sigma_B - 1}\frac{c_B}{p_B} = \frac{p_B(F+\varphi)N_B}{(p_B - c_B)L};$$

and

$$\left(1 - \frac{c_B}{p_B}\right)\left(1 - \frac{\sigma_B}{\sigma_B - 1}\frac{c_B}{p_B}\right) = \frac{(F+\varphi)N_B}{L}. \tag{35}$$

The solution of this equation with respect to p_B is given by

$$p_B = \frac{c_B}{1 - \frac{1}{2\sigma_B} - \sqrt{\frac{(F+\varphi)N_B}{L}\frac{\sigma_B - 1}{\sigma_B} + \frac{1}{4\sigma_B^2}}}.$$

The last equation is equivalent to (32). Substituting (32) to (34), we have

$$Q_B = \frac{F+\varphi}{c_B L}\left(\frac{2\sigma_B}{2\sigma_B - 1 - \sqrt{1 + 4\sigma_B(\sigma_B - 1)m}} - 1\right)^{-1}. \tag{36}$$

Equations (33) and (36) are equivalent. If we assigned the second root of Equation (35) to p_B, then the output found with (34) would be negative.

Now we derive Equation (16). We combine Equations (3) faced by small and big firms to:

$$\frac{u'(Q_B)}{u'(Q_S)} = \frac{p_B}{p_S}.$$

Then expressing p_B from the last equation and using Equation (6), we obtain

$$p_B = \frac{u'(Q_B)}{u'(Q_S)}\frac{c_S\sigma_S}{\sigma_S - 1}.$$

With this p_B, Equation (35) is transformed to (16). $\square$

Lemma 9. *Let $\sigma(Q)$ be a decreasing function. Then the equation*

$$Q_B = \frac{F+\varphi}{c_B L}\left(\frac{1}{1 - \frac{1}{2\sigma_B} - \sqrt{\frac{1}{4\sigma_B^2} + \left(1 - \frac{1}{\sigma_B}\right)m}} - 1\right)^{-1} \tag{37}$$

considered with respect to Q_B for any fixed $m \in [0, 1)$ has a unique solution. This solution $Q_B = Q_B(m)$ is a decreasing function of m.

Proof. We put $r(Q) = 1/\sigma(Q)$ and re-write Equation (37) as

$$Q_B = \frac{F+\varphi}{c_B L}\left(\frac{1}{1 - \frac{1}{2}r_B - \sqrt{\frac{1}{4}r_B^2 + (1 - r_B)m}} - 1\right)^{-1}.$$

We define an auxiliary function $h(r) = \frac{1}{2}r + \sqrt{\frac{1}{4}r^2 + (1-r)m}$, where $m \in [0,1)$ is a parameter, and establish its growth in r. Computing

$$h'(r) = \frac{1}{2} + \frac{\frac{1}{2r} - m}{2\sqrt{\frac{1}{4}r^2 + (1-r)m}},$$

we conclude that the inequality $h'(r) > 0$ is equivalent to the inequality

$$1 > \frac{m - r/2}{\sqrt{\frac{1}{4}r^2 + (1-r)m}}.$$

If $2m - r < 0$, then the last inequality holds. If $2m - r \leq 0$, the last inequality can be written as

$$\frac{1}{4}r^2 + (1-r)m > m^2 - mr + \frac{1}{4}r^2 \quad \text{or} \quad m > m^2,$$

which is evident. Since $\sigma' < 0$, it follows that $r' > 0$ and $h(r(Q))$ is an increasing function of Q. Then the right-hand side (rhs) of Equation (37) is a decreasing function of Q_B, whereas the left-hand side (lhs) is an increasing function. Since the lhs varies from 0 to $+\infty$ and the rhs is positive, their unique intersection exists. Investigating the rhs explicitly, we claim that it decreases in m. Then the solution $Q_B(m)$ decreases as a function of m. □

Lemma 10. *We assume that Q_B is defined as the solution of Equation* (37) *and is substituted into Equation* (16)*. Let $\sigma' < 0$. Then, a solution m of Equation* (16) *exists. Second, the difference of the right and left-hand sides of* (16) *changes its sign from plus to minus at the minimal m^*, which solves Equation* (16)*. If the solution m is unique, it also has this property.*

Proof. We plan to show that the rhs of (16) increases in m. The function $u'(Q_B)$ is positive and decreasing. Therefore, the second bracket in (16) is decreasing in Q_B. The first bracket is also decreasing because $u'(Q_B)(\sigma(Q_B) - 1)/\sigma(Q_B) = u'(Q_B)(1 - 1/\sigma(Q_B))$ decreases in Q_B. Thus, the rhs decreases in Q_B but increases in m.

The lhs of (16) increases from 0 to $+\infty$. If $m = 0$ then the equations determining Q_B and Q_S coincides and, as a result, $Q_B = Q_S$. Then $rhs(0) = (1 - c_B/c_S)^2 \in (0,1)$. Since the rhs is bounded by 1 from above, the intersection of the left and right-hand sides exists. Let m^* be the minimal m that satisfies (16). Then the difference of the right and left-hand sides of (16) changes its sign from plus to minus at m^*. □

Lemma 11. *Let $\tilde{Q}_B$ solves the equation $Q = (F + \varphi)(\sigma(Q) - 1)/(c_B L)$; $\tilde{q}_B = \tilde{Q}_B L$. Then $q_B < \tilde{q}_B$ in equilibrium.*

Proof. A simple algebra gives evidence that the product of the second and the third fraction on the right-hand side of Equation (33) is less than 1. As a result, from (33), it follows that $Q_B < (F + \varphi)(\sigma_B - 1)/(c_B L)$. According to Assumption 1, the right-hand side of the obtained inequality decreases in Q. Therefore, $Q_B < \tilde{Q}_B$. □

Proof of Proposition 1. Following the definition of equilibrium, we solve the consumer's optimization problem, then the firm's optimization problem and finally add all of the balances. Lemma 1 introduces a new variable, which is the Lagrange multiplier λ, and relates the optimal demand and the Lagrange multiplier to the other equilibrium variables, Equations (3) and (23).

We turn to the big firm's problem. The first order conditions are given by Equation (24), Lemma 2. In comparison with the first order conditions of the small firm's problem, the Gateaux derivative substitutes the partial derivative with respect to prices. This occurs because a big firm decides upon a range of prices represented by the function p_x.

In the next step, we vary the output with respect to the prices by proceeding with Equation (24) and excluding the Lagrange multiplier; see Lemma 4. Both operations are done due to Equation (3). The variation of the output involves the variation of the Lagrange multiplier performed in Lemma 3. This variation is zero for the small firm's optimization problem. Restricting ourselves to symmetrical solutions with respect to x, we reduce Equation (25), which obtained in Lemma 4, to (9); see Lemma 5.

Lemma 6 verifies the second order condition of the big firm's optimization problem and establishes that the prices given by (9) do maximize profits.

Taking into the consideration the free entry condition, Lemma 8 finds simple equations that the optimal demand and prices satisfy. At this moment, the system of the equilibrium equations is split into parts. Equation (14) explicitly determines the optimal demand Q_s. Indeed, the difference between the left-hand side $q_S = Q_S L$ and the right-hand side (which contains a decreasing function $\sigma(Q_S)$) increases and has a unique intersection with zero.

The system of Equations (33) and (16) implicitly determines the optimal demand Q_B for the varieties of big firms and the number m divided by the total population L. In fact, the question of the existence and uniqueness of equilibrium is reduced to the analysis of this system of the two equations. Lemmas 9 and (10) and establish that a solution to this system exists under Assumption (1). The uniqueness is evident under CES preferences. □

B Approximation to the Number of Big Firms

We start with the following technical result.

Lemma 12. *Let T_1 and T_2 denote the second and third fractions in Equation (33):*

$$T_1 = \frac{\sigma_B - \frac{1}{2} - \frac{1}{2}\sqrt{1 + 4\sigma_B(\sigma_B - 1)m}}{\sigma_B - 1} \qquad T_2 = \frac{2}{1 + \sqrt{1 + 4\sigma_B(\sigma_B - 1)m}} \tag{38}$$

Then the expansion of the product $T_1 T_2$ into series up the m^3-term is given by

$$T_1 T_2 \approx 1 - \sigma_B^2 m + 2\sigma_B^3(\sigma_B - 1)m^2 - \sigma_B^3(\sigma_B - 1)^2(4\sigma_B + 1)m^3. \tag{39}$$

Proof. We consequently expand the square root, fraction T_1, and fraction T_2 into series. The square root is:

$$\sqrt{1+4\sigma_B(\sigma_B-1)m} \approx 1+2\sigma_B(\sigma_B-1)m-2\sigma_B^2(\sigma_B-1)^2m^2+4\sigma_B^3(\sigma_B-1)^3m^3.$$

The factor T_1:

$$T_1 \approx 1-\sigma_B m+\sigma_B^2(\sigma_B-1)m^2-2\sigma_B^3(\sigma_B-1)^2m^3.$$

The factor T_2:

$$T_2 \approx \frac{1}{1+\sigma_B(\sigma_B-1)m-\sigma_B^2(\sigma_B-1)^2m^2+2\sigma_B^3(\sigma_B-1)^3m^3}.$$

$$\begin{aligned} T_2 = 1-\sigma_B(\sigma_B-1)m+\sigma_B^2(\sigma_B-1)^2m^2-2\sigma_B^3(\sigma_B-1)^3m^3+\sigma_B^2(\sigma_B-1)^2m^2-2\sigma_B^3(\sigma_B-1)^3m^3 \\ = 1-\sigma_B(\sigma_B-1)m+2\sigma_B^2(\sigma_B-1)^2m^2-4\sigma_B^3(\sigma_B-1)^3m^3. \end{aligned}$$

Then Equation (39) gives the product T_1T_2. □

Proof of Proposition 2. The proof is based on the expansion of the right-hand side of Equation (16) into series. We seek the function m as a series with respect to ε^γ, where $\varepsilon = 1-c_B/c_S$ and γ is an appropriate exponent. The following direct computation gives evidence that m is the leading term in the right-hand side of Equation (16). Then m goes off both sides, and the leading terms remaining in the equation are m^2 and ε. This suggests that $\gamma = 1/2$ and the expansion of m is in powers of $\varepsilon^{1/2}$. We will justify that the representation $m = B\varepsilon^{1/2}$ required for Equation (21) involves the expansion of (16) to order x^2. A more detailed expansion to order x^3 enables us to obtain the representation $m = B\varepsilon^{1/2}+C\varepsilon$ and eventually Equation (20). Under unspecified preferences, the expansion to order x^3 is too complicated to be exposed with simple terms. We turn to algebra, dropping the most computational part.

Initially, we perform the expansion under the CES setting and later we repeat the arguments under unspecified preferences. Under the CES setting, $u'(Q_B)/u'(Q_S) = (Q_S/Q_B)^{1/\sigma}$, we are able to expand the right-hand side up to the second order term using the form:

$$\frac{u'(Q_S)}{u'(Q_B)} = 1-\left(\frac{c_B}{c_S}\right)^{\frac{\sigma-1}{\sigma}}\left(1+\frac{\varphi}{F}\right)^{-\frac{1}{\sigma}}(T_1T_2)^{\frac{1}{\sigma}}.$$

By using (39), we have

$$(T_1T_2)^{\frac{1}{\sigma}} = 1-\sigma m+\frac{3}{2}\sigma^2(\sigma-1)m^2-\sigma^2(\sigma-1)^2(2\sigma+1)m^3.$$

Now we return to Equation (16), written in the form:

$$m = \left(1-\left(\frac{c_B}{c_S}\right)^{\frac{\sigma-1}{\sigma}}\left(1+\frac{\varphi}{F}\right)^{-\frac{1}{\sigma}}(T_1T_2)^{\frac{1}{\sigma}}\right)\left(1-\frac{\sigma-1}{\sigma}\left(\frac{c_B}{c_S}\right)^{\frac{\sigma-1}{\sigma}}\left(1+\frac{\varphi}{F}\right)^{-\frac{1}{\sigma}}(T_1T_2)^{\frac{1}{\sigma}}\right) \quad (40)$$

Put,

$$\tilde{\varepsilon} = 1 - \left(\frac{c_B}{c_S}\right)^{\frac{\sigma-1}{\sigma}} \left(1 + \frac{\varphi}{F}\right)^{-\frac{1}{\sigma}}.$$

Then Equation (40) becomes

$$0 = \frac{\tilde{\varepsilon}}{\sigma} - \frac{1}{2}\sigma(\sigma-1)m^2 + (2\sigma-3)\tilde{\varepsilon}m - \sigma(\sigma-1)^3m^3.$$

We seek m as a function of $\tilde{\varepsilon}$ in a form $m = B\tilde{\varepsilon}^\gamma + C\tilde{\varepsilon}^{2\gamma} + \ldots$, $\gamma > 0$, $B, C \in \mathbb{R}$, $B \neq 0$. Then $\gamma = 1/2$, $m \approx B\sqrt{\tilde{\varepsilon}} + C\tilde{\varepsilon}$, and the factors B and C can be found equalizing the coefficients at the same powers of $\tilde{\varepsilon}$. Equalizing the coefficients multiplied by $\tilde{\varepsilon}$, we have

$$\frac{1}{\sigma} - \frac{1}{2}\sigma(\sigma-1)B^2 = 0,$$

and

$$B = \frac{1}{\sigma}\sqrt{\frac{2}{\sigma-1}}.$$

Equalizing the coefficients multiplied by $\tilde{\varepsilon}^{3/2}$, we have

$$-\frac{1}{2}\sigma(\sigma-1)2BC + (2\sigma-3)B = \sigma(\sigma-1)^3B^3.$$

This turns to

$$C = \frac{\sigma-2}{\sigma^2(\sigma-1)}.$$

We conclude that

$$m \approx \frac{1}{\sigma}\sqrt{\frac{2}{\sigma-1}}\sqrt{\tilde{\varepsilon}} + \frac{\sigma-2}{\sigma^2(\sigma-1)}\tilde{\varepsilon}.$$

Let $\varepsilon = 1 - c_B/c_S$. Then the expression for $\tilde{\varepsilon}$ is expanded into $\tilde{\varepsilon} \approx \frac{\sigma-1}{\sigma}\varepsilon + \frac{\varphi}{\sigma F}$, and

$$m \approx \frac{\sqrt{2}}{\sigma^{3/2}(\sigma-1)^{1/2}}\left((\sigma-1)\varepsilon + \frac{\varphi}{F}\right)^{1/2} + \frac{\sigma-2}{\sigma^3(\sigma-1)}\left((\sigma-1)\varepsilon + \frac{\varphi}{F}\right). \tag{41}$$

Inverting the last equation, we have:

$$m^{-1} \approx \frac{\sigma^{3/2}(\sigma-1)^{1/2}}{\sqrt{2}}\left((\sigma-1)\Big(1 - \frac{c_B}{c_S}\Big) + \frac{\varphi}{F}\right)^{-1/2} - \frac{\sigma}{2} + 1.$$

Now Equation (20) follows from (10).

We return to Equation (21), intending to solve Equation (16) and prove the approximation

$$m^2 \approx \frac{2r_u^3(2-r_{u'})}{1-r_u}\left(\varepsilon + \frac{r_u\varphi}{(1-r_u)F}\right). \tag{42}$$

In Equation (42) and in the rest of the proof, the functions r_u and $r_{u'}$ are taken at the point Q_B. Auxiliary computation gives evidence that

$$(r_u)' = \frac{r_u}{Q}(1 + r_u - r_{u'}), \quad (r_{u'})' = \frac{r_{u'}}{Q}(1 + r_{u'} - r_{u''}), \tag{43}$$

$$(r_u)'' = \frac{r_u}{Q^2}(2r_u + 2r_u^2 - 2r_{u'} - 3r_ur_{u'} + r_{u'}r_{u''}), \tag{44}$$

The following expansion into series holds:

$$\frac{\sigma_S - 1}{\sigma_B - 1} \approx 1 - \frac{1 + r_u - r_{u'}}{1 - r_u} \left(\frac{Q_S}{Q_B} - 1 \right) + \frac{1 + r_u - r_{u'} + r_{u'}^2 - \frac{1}{2} r_u r_{u'} - \frac{1}{2} r_{u'} r_{u''}}{1 - r_u} \left(\frac{Q_S}{Q_B} - 1 \right)^2. \tag{45}$$

We claim that the equation

$$\frac{\sigma_S - 1}{\sigma_B - 1} \frac{c_B}{c_S} = \frac{Q_S}{Q_B} T_1 T_2 \left(1 + \frac{\varphi}{F} \right), \tag{46}$$

when considered with respect to Q_S/Q_B, has the solution

$$\frac{Q_S}{Q_B} - 1 \approx -\frac{1 - r_u}{2 - r_{u'}} \left(\varepsilon - \frac{\varphi}{F} \right) + \frac{1 - r_u}{(2 - r_{u'}) r_u^2} m + K_2 m^2 \tag{47}$$

up to $\bar{o}(m^2)$, where

$$K_2 = \frac{1 + r_u - r_{u'} + r_{u'}^2 - \frac{1}{2} r_u r_{u'} - \frac{1}{2} r_{u'} r_{u''}}{2 - r_{u'}} \frac{(1 - r_u)^2}{(2 - r_{u'})^2 r_u^4} - \frac{2(1 - r_u)^2}{(2 - r_{u'}) r_u^4} + \frac{(1 - r_u)^2}{(2 - r_{u'})^2 r_u^4}.$$

We evaluate the ratio u''_S/u''_B as

$$\frac{u''_S}{u''_B} \approx 1 - r_{u'} \left(\frac{Q_S}{Q_B} - 1 \right) + \frac{1}{2} r_{u'} r_{u''} \left(\frac{Q_S}{Q_B} - 1 \right). \tag{48}$$

According to (15), (14), (38), and the definition of σ, Equation (16) turns into

$$m = \left(1 - \left(\frac{Q_S}{Q_B} \right)^2 \frac{u''_S}{u''_B} \left(1 - \frac{\varphi}{F} \right) T_1 T_2 \right) \left(1 - \frac{\sigma_B - 1}{\sigma_B} \left(\frac{Q_S}{Q_B} \right)^2 \frac{u''_S}{u''_B} \left(1 - \frac{\varphi}{F} \right) T_1 T_2 \right) \tag{49}$$

Inside this proof, S_1 and S_2 denote the first and the second brackets of the right-hand side of Equation (49). We simplify the first bracket (a routine algebra is skipped here):

$$S_1 \approx (1 - r_u)\varepsilon + \frac{r_u \varphi}{F} + \frac{m}{r_u} + \left(\frac{(1 - r_u)}{r_u^3} + \frac{(1 - r_u)^2}{2(2 - r_{u'}) r_u^3} m^2 \right).$$

The second bracket S_2 is $S_2 = 1 - (1 - r_u)(1 - S_1) = r_u + (1 - r_u) S_1$. Since $S_1^2 = m^2/r_u^2 + \bar{o}(m^2)$, it follows that $S_1 S_2 \approx r_u S_1 + (1 - r_u) S_1^2$ and Equation (49) is reduced to

$$m \approx r_u (1 - r_u)\varepsilon + \frac{r_u^2 \varphi}{F} + m - r_u \left(\frac{(1 - r_u)}{r_u^3} + \frac{(1 - r_u)^2}{2(2 - r_{u'}) r_u^3} \right) m^2 - \frac{(1 - r_u) m^2}{r_u^2}.$$

Simplifying, we get (42). □

Proof of Proposition 3. If r_u increases and $r_{u'}$ decreases, then the fraction $(1 - r_u)/(2 - r_{u'})$ decreases and the right-hand side of Equation (42) increases in Q_B.

Let the population increase from L_1 to $L_2 > L_1$. We turn to Equation (33) with the "old" $m_1 = m(L_1)$ but the new $L = L_2$. Due to the change in L, the right-hand side of (33) decreases, and the solution of this equation also decreases: $Q_B(m_1, L_2) < Q_B(m_1, L_1)$. Substituting this $Q_B(m_1, L_2)$ into Equation (42) we find that its right-hand side (rhs), as an increasing function in Q_B, has decreased and $m_1 > rhs\big(Q_B(m_1, L_2)\big)$. Since $Q_B(\cdot, L_2)$ decreases with the first argument (see Lemma 9), rhs also decreases in m. Therefore, changing m from m_1 downward, we decrease the left-hand side m^2 of (42) and increase the right-hand side. Since $0 < rhs\big(Q_B(0, L_2)\big)$, there is a new solution m_2 located to the left of m_1. The number of big firms, being inverse to m, increases. □

References

A. Alesina and E. Spolaore. On the number and size of nations. *The Quarterly Journal of Economics*, 112(4):1027–1056, 1997.

K. Behrens and Y. Murata. Trade, competition, and efficiency. *Journal of International Economics*, 87(1):1–17, 2012.

A. B. Bernard, S. J. Redding, and P.-K. Schott. Comparative Advantage and Heterogeneous Firms. *Review of Economic Studies*, 74:31–66, 2007.

A. B. Bernard, J. B. Jensen, S. J. Redding, and P. K. Schott. The empirics of firm heterogeneity and international trade. *Annu. Rev. Econ.*, 4(1):283–313, 2012.

P. Bolton and G. Roland. The breakup of nations: a political economy analysis. *The Quarterly Journal of Economics*, 112(4):1057–1090, 1997.

C. d'Aspremont, R. Ferreira, and L.-A. Gérard-Varet. On monopolistic competition and involuntary unemployment. *The Quarterly Journal of Economics*, 105(4):895–919, 1990.

C. d'Aspremont, R. Ferreira, and L.-A. Gérard-Varet. On the dixit-stiglitz model of monopolistic competition. *The American Economic Review*, pages 623–629, 1996.

H. Demsetz. Industry structure, market rivalry, and public policy. *The Journal of Law and Economics*, 16(1):1–9, 1973.

S. Dhingra. Trading away wide brands for cheap brands. *The American Economic Review*, 103 (6):2554–2584, 2013.

A. Dixit. A model of duopoly suggesting a theory of entry barriers. *J. Reprints Antitrust L. & Econ.*, 10:399, 1979.

K. Dixit and E. Stiglitz. Monopolistic Competition and Optimum Product Diversity. *The American Economic Review*, 67:297–308, 1977.

C. Eckel and J. P. Neary. Multi-product firms and flexible manufacturing in the global economy. *The Review of Economic Studies*, 77(1):188–217, 2010.

F. Etro. Stackelberg competition with endogenous entry. *Economic Journal*, 118:1670–1697, 2008.

R. Feenestra and H. Ma. Optimal choice of product scope for multiproduct firms under monopolistic competition, 2007.

J. J. Gabszewicz and J.-P. Vial. Oligopoly "a la cournot" in a general equilibrium analysis. *Journal of Economic Theory*, 4(3):381–400, 1972. doi: https://doi.org/10.1016/0022-0531(72)90129-9.

C. J. Hottman, S. J. Redding, and D. E. Weinstein. Quantifying the sources of firm heterogeneity. *The Quarterly Journal of Economics*, 131(3):1291–1364, 2016.

S. Kokovin, M. Parenti, J-F. Thisse, and P. Ushchev. On the dilution of market power, 2017.

P. Krugman. Increasing returns, monopolistic competition and international trade. *Journal of International Economics*, 9:469–479, 1979.

G. Lawrence. *Capitalism and the countryside: The rural crisis in Australia*. Pluto Press, 1987.

N. G. Mankiw. *Principles of Economics, 4th edition*. Thomson, 2007.

T. Mayer, M. J. Melitz, and G.I.P. Ottaviano. Market size, competition, and the product mix of exporters. *The American Economic Review*, 104(2):495–536, 2014.

M. J. Melitz. The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity. *Econometrica*, 71:1695–1725, 2003.

M. J. Melitz and S. J. Redding. Heterogeneous Firms and Trade. *NBER Working Paper No. 18652*, 2012. URL `http://www.nber.org/papers/w18652`.

J. P. Neary. Globalization and market structure. *Journal of the European Economic Association*, 1(2-3):245–271, 2003.

J. P. Neary. Two and a half theories of trade. *The World Economy*, 33(1):1–19, 2010.

G. I. Ottaviano, T. Tabuchi, and J.-F. Thisse. Agglomeration and Trade Revisited. *International Economic Review*, 43:409–436, 2002.

M. Parenti. Large and small firms in a global market: David vs. goliath. *Journal of International Economics*, 2017.

M. E. Porter. *Competitive strategy: Techniques for analyzing industries and competitors*. Simon and Schuster, 2008.

S. J. Redding. Theories of heterogeneous firms and trade. *Annu. Rev. Econ.*, 3(1):77–105, 2011.

K.-I. Shimomura and J.-F. Thisse. Competition among the big and the small. *The Rand Journal of Economics*, 43(2):329–347, 2012.

P. Ushchev. Multi-product firms in monopolistic competition: The role of scale-scope spillovers. *Research in Economics*, 71(4):675–689, 2017.

C. Uslay, Z. A. Altintig, and R. D. Winsor. An empirical examination of the “rule of three”: strategy implications for top management, marketers, and investors. *Journal of Marketing*, 74(2):20–39, 2010.

X. Yang and B. J. Heijdra. Monopolistic competition and optimum product diversity: Comment. *The American Economic Review*, 83(1):295–301, 1993.